Exercise Set 2: Real Number Proving

These exercises are intended to illustrate basic techniques to prove formulas involving real numbers. The PVS file `exercises/realproving.pvs` support these exercises.

1. A googol is the number 10^{100} and a googolplex is the number 10^{googol}.

   Problem: Prove that a googol plex is strictly greater than the multiplication of two googols. Lemma `googolplex_gt_googol2` specifies this statement in PVS.

   Hint: The following lemmas are defined in the PVS prelude.

   ```
   expt_plus: LEMMA n0x^(i + j) = n0x^i * n0x^j

   both_sides_expt_gt1_gt: LEMMA gt1x ^ i > gt1x ^ j IFF i > j
   ```

   Use both lemmas to reduce the problem to proving that $10^{100} > 200$. At this point, prove that $100^{100} > 10^3$, using lemma `both_sides_expt_gt1_gt`, and that $10^3 > 200$, using `(grind)`.

2. **Problem:** Prove that for $x, y \in \mathbb{R}$, $y(1-x)(1-x) \leq 0$ if $y \leq 0$.

3. **Problem:** Prove that for a non-negative real number x less than or equal to 1, $\sqrt{1-x} \leq 1$. Lemma `simplesqrt1` specifies this statement, where `sqrt` is the square root function defined in `reals@sqrt`.

   Hint: Transforms the formula $\sqrt{1-x} \leq 1$ into the formula $\sqrt{1-x} \times \sqrt{1-x} \leq 1$ using, for example, the proof rule `mult-ineq`. Then, `(assert)`.

4. **Problem:** Prove lemmas `p1`, `p2`, and `p3` in `exercises/realproving.pvs` using strategies in `Manip` and `Field`.