# Flight Schedule Database Example

by

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# Flight Schedule Example

## Requirements for an Airport Flight Schedule Database

- The flight schedule database shall store the scheduling information associated with all departing and arriving flights. In particular the database shall contain:
  - departure time and gate number
  - arrival time and gate number
  - route (i.e. navigation way points)

  for each arriving and departing flight.
- There shall be a way to retrieve the scheduling information given a flight number.
- It shall be possible to add and delete flights from the database.

## Formal Requirements Specification

- How do we represent the flight schedule database mathematically?

  1. a set of ordered pairs of flight numbers and schedules. Adding and deleting entries via set addition and deletion

  2. function whose domain is all possible flight numbers and range is all possible schedules. Adding and deleting entries via modification of function values.

  3. function whose domain is only flight numbers currently in database and range is the schedules. Adding and deleting entries via modification of the function domain and values.

Note: The choice between these is strongly influenced by the verification system used.

## Getting Started

<u>**Let's start with approach 2:**</u>

> **function whose domain is all possible flight numbers and range is all possible schedules. Adding and deleting entries via modification of function values.**

**In traditional mathematical notation, we would write:**

$$
\begin{aligned}
\textbf{Let } N &= \textbf{set of flight numbers} \\
S &= \textbf{set of schedules} \\
D &: N \longrightarrow S
\end{aligned}
$$

**where $D$ represents the database and $S$ represents all of the schedule information.**

**Note that the details have been *abstracted away*. This is one of the most important steps in producing a good formal specification.**

## Specifying the Flight Schedule Database

$$D : N \longrightarrow S$$

**How do we indicate that we do not have a flight schedule for all possible flight numbers?**

We declare a constant of type $S$, say "$u_o$", that indicates that there is no flight scheduled for this flight number.

**Now can define an empty database. In traditional notation, we would write:**

$$empty\_database : N \longrightarrow S$$

$$empty\_database(flt) \equiv u_o$$

$$\forall \; flt \in N$$

## Accessing an Entry

**Let** $N =$ **set of flight numbers**

$S =$ **set of schedules**

$D =$ **set of functions** $: N \longrightarrow S$

$\forall d \in D$ **and** $flt \in N$.

$$find\_schedule : D \times N \longrightarrow S$$

$$find\_schedule(d, flt) = d(flt)$$

**Note that** $find\_schedule$ **is a higher-order function since its first argument is a function.**

## Specifying Adding/Deleting an Entry

**Let** $N =$ **set of flight numbers**

$S =$ **set of schedules**

$D : N \longrightarrow S$

$u_o \in S$

$D =$ **set of functions** $: N \longrightarrow S$

$\forall d \in D, \ \ \forall flt \in N, \ \ \forall sched \in S$

$$add\_flight : D \times N \times S \longrightarrow D$$

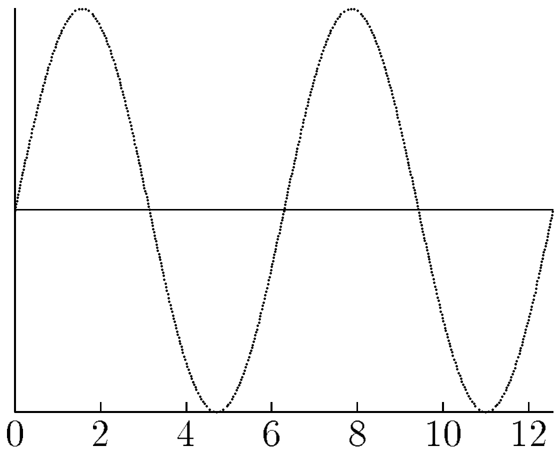
$$add\_flight(d, flt, sched)(x) = \begin{cases} d(x) & \textbf{if } x \neq flt \\ sched & \textbf{if } x = flt \end{cases}$$

$$delete\_flight : D \times N \longrightarrow D$$

$$delete\_flight(d, flt)(x) = \begin{cases} d(x) & \textbf{if } x \neq flt \\ u_o & \textbf{if } x = flt \end{cases}$$

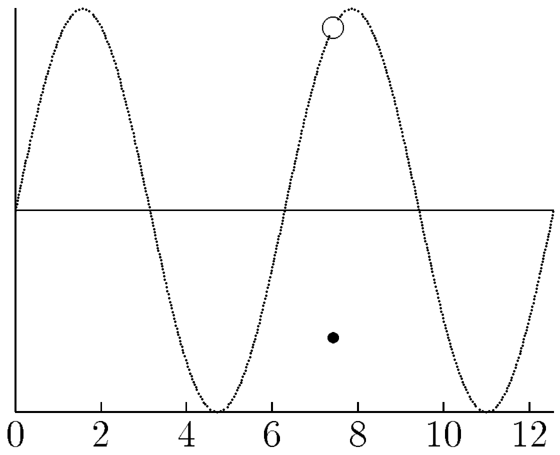
# The WITH Notation

**sin(x):**




$$\textbf{sin(x) WITH [7.4 := -0.60 ]} = \begin{cases} \textbf{-0.60} & \textbf{if } x = \textbf{7.4} \\ sin(x) & \textbf{otherwise} \end{cases}$$

**sin(x) WITH [7.4 := -0.60 ]**



# Complete Spec (Omitting Function Signatures)

**Let** $N =$ **set of flight numbers**

$S =$ **set of schedules**

$D =$ **set of functions** $: N \longrightarrow S$

$\forall d \in D, \;\; \forall flt \in N, \;\; \forall sched \in S$

$$find\_schedule(d, flt) = d(flt)$$

$$add\_flight(d, flt, sched)(x) = d \textbf{ WITH } [flt := sched]$$

$$delete\_flight(d, flt)(x) = d \textbf{ WITH } [flt := u_o]$$

**Can test spec with some putative theorems:**

**LEMMA** (*putative 1*) : $find\_schedule(add\_flight(d, flt, sched), flt) = sched$

**LEMMA** (*putative 2*) : $delete\_flight(add\_flight(d, flt, sched), flt) = d$

## Attempted Verification Of Putative 2 Reveals a Problem

**LEMMA** (*putative 2*): $delete\_flight(add\_flight(d, flt, sched), flt) = d$
**Proof:**

$$delete\_flight(add\_flight(d, flt, sched), flt) =$$

$$delete\_flight(d \textbf{ WITH } [flt := sched]) =$$

$$d \textbf{ WITH } [flt := sched] \textbf{ WITH } [flt := u_o] =$$

$$d \textbf{ WITH } [flt := u_o] = ??$$

**But there is no way to reach** $d$**, because**

$$d \textbf{ WITH } [flt := u_o] \neq d$$

**unless** $d(flt) = u_o$**.**

**This is only true if the** $flt$ **is currently not scheduled in the flight database.**

## Verification Reveals Oversight

- **We realize that we only want to add a flight with flight number $flt$, if one is not already in the database.**
- **If $flt$ is already in the database, we probably need the capability to change it.**

**Thus, we modify $add\_flight$ and create a new function $change\_flight$:**

## Verification Reveals Oversight (Cont.)

**Let** $N =$ **set of flight numbers**

$S =$ **set of schedules**

$D =$ **set of functions** $: N \longrightarrow S$

$\forall d \in D, \;\; \forall flt \in N, \;\; \forall sched \in S$

$scheduled?(d, flt) : boolean = d(flt) \neq u_o$

$add\_flight(d, flt, sched) =$

$\quad$ **IF** $scheduled?(d, flt)$ **THEN** $d$

$\quad$ **ELSE** $d$ **WITH** $[flt := sched]$ **ENDIF**

$change\_flight(d, flt, sched) =$

$\quad$ **IF** $scheduled?(d, flt)$ **THEN** $d$ **WITH** $[flt := sched]$

$\quad$ **ELSE** $d$ **ENDIF**

# Putative 2 Proof After Correction

**LEMMA** (*putative 2*)**: NOT** $scheduled?(d, flt) \supset$
$$delete\_flight(add\_flight(d, flt, sched), flt) = d$$

**Proof:**

$$delete\_flight(add\_flight(d, flt, sched), flt)$$

$$= delete\_flight(\ \textbf{IF}\ scheduled?(d, flt)\ \textbf{THEN}\ d$$
$$\textbf{ELSE}\ d\ \textbf{WITH}\ [flt := sched]\ \textbf{ENDIF}\ )$$

$$= delete\_flight(d\ \textbf{WITH}\ [flt := sched])$$

$$= d\ \textbf{WITH}\ [flt := sched]\ \textbf{WITH}\ [flt := u_o]$$

$$= d\ \textbf{WITH}\ [flt := u_o]$$

$$= d \qquad \textbf{(because NOT}\ scheduled?(d, flt) \supset d(flt) = u_o\ \textbf{)}$$

## A Minor Problem

**To check our new function `schedule?` we postulate the following putative theorem:**

**SchedAdd: LEMMA** $scheduled?(add\_flight(d, flt, sched), flt)$

**Proof:**

$$scheduled?(add\_flight(d, flt, sched)) =$$

$$scheduled?(\ \textbf{IF}\ scheduled?(d, flt)\ \textbf{THEN}\ d$$

$$\textbf{ELSE}\ d\ \textbf{WITH}\ [flt := sched]\ \textbf{ENDIF}\ =$$

$$\textbf{IF}\ d(flt) \neq u_o\ \textbf{THEN}\ d(flt) \neq u_o$$

$$\textbf{ELSE}\ d\ \textbf{WITH}\ [flt := sched](flt) \neq u_o\ \textbf{ENDIF}\ =$$

$$d\ \textbf{WITH}\ [flt := sched](flt) \neq u_o$$

$$sched \neq u_o$$

**which is not provable because nothing prevents** $sched = u_o$**.**

## A Minor Problem Repaired

**We then realize that our specification does not rule out the possibility of assigning a "$u_o$" schedule to a real flight**

**Let** $N =$ **set of flight numbers**

$S =$ **set of schedules**

$S^* =$ **set of schedules not including** $u_o$

$D =$ **set of functions** $: N \longrightarrow S$

$\forall d \in D, \ \ \forall flt \in N, \ \ \forall sched \in S^*$

$$find\_schedule : D \times N \longrightarrow S$$

$$add\_flight : D \times N \times S^* \longrightarrow D$$

$$change\_flight : D \times N \times S^* \longrightarrow D$$

$$delete\_flight : D \times N \longrightarrow D$$

**This type of trivial problem is usually not manifested until when one attempts a mechanical (i.e. level 3) verification.**

## Another Example of a Putative Theorem

$$
\begin{array}{ll}
 & (\forall i : flt_i \neq flt) \wedge \\
 & \\
 & find\_schedule(d_0, flt) = sched \wedge \\
 & d_1 = add\_flight(d_0, flt_1, sched_1) \wedge \\
 & d_2 = add\_flight(d_1, flt_2, sched_2) \wedge \\
 & \quad \cdot \qquad\qquad\qquad\qquad \cdot \\
 & \quad \cdot \qquad\qquad\qquad\qquad \cdot \\
 & \quad \cdot \qquad\qquad\qquad\qquad \cdot \\
 & d_n = add\_flight(d_{n-1}, flt_n, sched_n) \\
\supset & \\
 & find\_schedule(d_n, flt) = sched
\end{array}
$$

- **Formal methods can establish that even in the presence of an *arbitrary* number of operations a property holds.**
- **Testing can never establish this.**

## Some Observations

- Our specification is abstract. The functions are defined over infinite domains.
- As one translates the requirements into mathematics, many things that are usually left out of English specifications are explicitly enumerated.
- The formal process exposes ambiguities and deficiencies in the requirements.
- Putative theorem proving and scrutiny reveals deficiencies in the formal specification.

# PVS Spec

```
flight_sched3: THEORY
BEGIN

  N : TYPE                    % flight numbers
  S : TYPE                    % schedules
  D : TYPE = [N -> S]         % flight database

  u0: S                       % unscheduled

  S_good  : TYPE = {sched: S | sched /= u0}

  flt  : VAR N
  d    : VAR D
  sched : VAR S_good

  emptydb(flt): S = u0

  find_schedule(d, flt): S = d(flt)

  scheduled?(d,flt): boolean = d(flt) /= u0
```

```
  add_flight(d, flt, sched): D =
       IF scheduled?(d,flt) THEN d
       ELSE d WITH [flt := sched] ENDIF

  change_flight(d, flt, sched): D =
       IF scheduled?(d,flt) THEN d WITH [flt := sched]
       ELSE d ENDIF

  delete_flight(d, flt): D = d WITH [flt := u0]


  putative2 : LEMMA NOT scheduled?(d,flt) IMPLIES
                      delete_flight(add_flight(d,flt,sched),flt) = d

  SchedAdd  : LEMMA scheduled?(add_flight(d,flt,sched),flt)


END flight_sched3
```

## Introduction to a PVS Proof

- **Illustrative proof**
- **The single command `GRIND` proves it automatically**

```
putative2 :

  |-------
{1}   (FORALL (d: D, flt: N, sched: S_good):
         NOT scheduled?(d, flt)
           IMPLIES delete_flight(add_flight(d, flt, sched), flt) = d)

Rule? (SKOSIMP*)
Repeatedly Skolemizing and flattening,
this simplifies to:
putative2 :

  |-------
{1}   scheduled?(d!1, flt!1)
{2}   delete_flight(add_flight(d!1, flt!1, sched!1), flt!1) = d!1

Rule? (EXPAND "add_flight" )
Expanding the definition of add_flight,
```

```
this simplifies to:
putative2 :

  |-------
[1]   scheduled?(d!1, flt!1)
{2}   delete_flight(IF scheduled?(d!1, flt!1) THEN d!1
                 ELSE d!1 WITH [flt!1 := sched!1]
                 ENDIF,
                 flt!1)
        = d!1

Rule? (LIFT-IF )
Lifting IF-conditions to the top level,
this simplifies to:
putative2 :

  |-------
[1]   scheduled?(d!1, flt!1)
{2}   IF scheduled?(d!1, flt!1) THEN delete_flight(d!1, flt!1) = d!1
      ELSE delete_flight(d!1 WITH [flt!1 := sched!1], flt!1) = d!1
      ENDIF

Rule? (ASSERT)
Simplifying, rewriting, and recording with decision procedures,
```

```
this simplifies to:
putative2 :

  |-------
[1]   scheduled?(d!1, flt!1)
{2}   delete_flight(d!1 WITH [flt!1 := sched!1], flt!1) = d!1

Rule? (EXPAND "delete_flight" )
Expanding the definition of delete_flight,
this simplifies to:
putative2 :

  |-------
[1]   scheduled?(d!1, flt!1)
{2}   d!1 WITH [flt!1 := sched!1] WITH [flt!1 := u0] = d!1

Rule? (EXPAND "scheduled?" )
Expanding the definition of scheduled?,
this simplifies to:
putative2 :

  |-------
{1}   d!1(flt!1) /= u0
[2]   d!1 WITH [flt!1 := sched!1] WITH [flt!1 := u0] = d!1
```

```
Rule? (assert)
Simplifying, rewriting, and recording with decision procedures,
Q.E.D.


Run time  = 1.16 secs.
Real time = 61.49 secs.

Wrote proof file /airlab/home/rwb/fm/wkshp/pvs/flight_sched3.prf
NIL
>
```

# New Requirement

**"Two flights are not to be scheduled at the same gate at the same time!"**

**Introduce refinement of schedule:**

```
N   : TYPE
Date_and_time: TYPE
Gate_nums: TYPE
A_or_D: TYPE = (arriving,departing)
Way: TYPE

S: TYPE = [# % RECORD
          departure_tm: Date_and_time,
          arrival_tm: Date_and_time,
          dep_gate: Gate_nums,
          arr_gate: Gate_nums,
          arr_or_dep: A_or_D,
          nav_route: Way
          #] % END RECORD
```

# Simplified Problem

**Often it is useful to solve a simplified problem before you tackle the big problem. So let's only work with departing flights:**

```
N  : TYPE
Date_and_time: TYPE
Gate_nums: TYPE

S: TYPE = [# % RECORD
          departure_tm: Date_and_time,
          dep_gate: Gate_nums,
          #] % END RECORD
```

**The requirement states that "two flights are not to be scheduled at the same gate at the same time":**

```
same_time(sched1, sched2): boolean
```

# New Requirement Continued

**We also need to introduce concept of "scheduled at the same gate at the same time":**

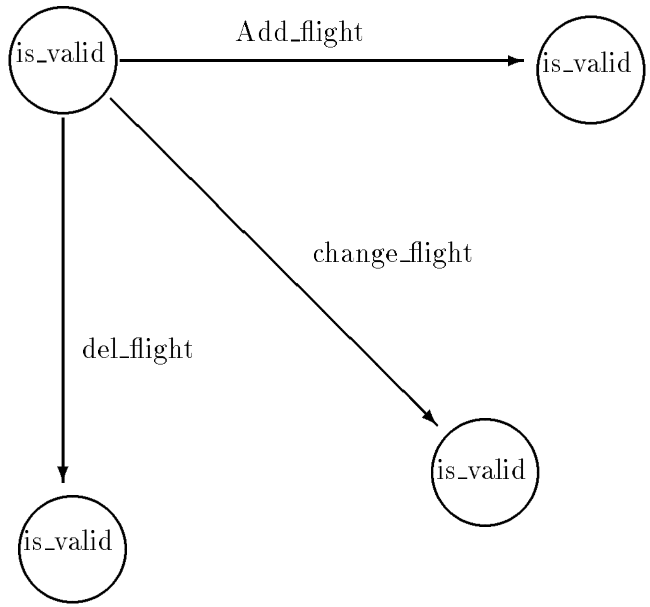
```
overlapped(sched1,sched2): boolean = dep_gate(sched1) = dep_gate(sched2)
                                     AND same_time(sched1,sched2)
```

**We would like to establish that the operations on the database will never result in an** `overlapped` **situation.**

**In other words, we want to establish an** ***invariant*****:**

```
is_valid(d: D): boolean = (FORALL (flt1,flt2: N): flt1 /= flt2 AND
           scheduled?(d,flt1) AND scheduled?(d,flt2) IMPLIES
                NOT overlapped(find_schedule(d,flt1), find_schedule(d,flt2)))
```

## Database System as a State Machine




**Need to establish that all of the "operations" maintain the invariant. For example,**

```
add_flight_is_valid: LEMMA
      (FORALL (d: D, flt: N, sched: S_good):
            is_valid(d) IMPLIES is_valid(add_flight(d,flt,sched)));
```

**Of course, `add_flight` must be modified to insure that this is true:**

## Add_flight Modified To Maintain Invariant

```
gate_in_use_at_time(d,sched): boolean =
       (EXISTS flt: scheduled?(d,flt) AND overlapped(sched,d(flt)))

add_flight(d, flt, sched): D =
     IF scheduled?(d,flt) OR gate_in_use_at_time(d,sched) THEN d
     ELSE d WITH [flt := sched]
     ENDIF
```

**Thus, we have modeled the database as a finite state machine and the functions** `add_flight`**,** `change_flight`**, and** `delete_flight` **are operations on the state machine.**

## State Machines and PVS Type System

**By creating a predicate subtype of the type D:**

```
Valid_db:     TYPE = {d: D | is_valid(d)}
```

**and modifying the signatures of** `add_flight`, `change_flight`, **and** `delete_flight`, **e.g.**

```
add_flight(vd: Valid_db, flt, sched): Valid_db =
     IF scheduled?(vd,flt) OR gate_in_use_at_time(vd,sched) THEN vd
     ELSE vd WITH [flt := sched]
     ENDIF
```

**PVS will automatically generate the "invariant" lemmas that must proved (called TCC's).**

- **is just a particular case of the more general TCC mechanism**
- **illustrates how a mechanized specification language can provide much stronger typechecking than traditional programming languages**

## Conclusions

- **With formal methods a clear, unambiguous, abstract specification can be constructed.**
- **Mechanized formal methods allows you can CALCULATE (prove) whether the specification has certain properties.**
- **These calculations can be done early in the lifecycle on abstract descriptions.**
- **And they can cover ALL the case,.**