

## Exercise 4: First Order Proving

Part of this exercise continues with variations on the theories from Exercise 3. The remainder is based on a logic puzzle example.

1. During the first part of Exercise 4, start by proving the quantified formulas in file `pred.basic.pvs`. Use the same techniques as in Exercise 3, except add in the quantifier rules (`skolem`, `inst` and their variants).
2. During the second part of Exercise 4, prove the first order version of FLT found in theory `FLT_lite_pred`. The theorem is called `FLT_very_lite`. Use the same techniques as in Exercise 3, except add in the quantifier rules. As before, do not try to prove any of the supporting lemmas.
3. The last part of the exercise is contained in the file `caucus.pvs` and is based on the formalization of a logic puzzle. The puzzle is described on the next page of these instructions. Relationships among the puzzle entities are expressed using types and functions. Clues from the puzzle are encoded as axioms while the puzzle solution is encoded as a set of lemmas.

Study the puzzle and its formalization in PVS. Your task is to prove the lemmas, which can be done using the commands `split`, `flatten`, `lemma`, `typepred`, `assert` and `inst` (and their variants). Hints are provided for each lemma to guide the proof process.

## The Advanced Aerospace Caucus<sup>1</sup>

Several senators have banded together to form the Advanced Aerospace Caucus, a group dedicated to promoting extremely ambitious aerospace projects. Caucus members hope to fund several projects and invigorate the economies of their home states. Each senator has been known to give long, boring speeches on the Senate floor in support of his or her pet project. Given the clues that follow, determine the project each senator is promoting and how much research funding is being sought for each.

1. Senator Tedi-  
us has a budget request greater than that of Senator Loquacius, whose budget figure is seven million dollars more than the cost of the supersonic helicopter project.
2. The cost of the flying submarine project is not equal to \$58M, but it is less than the cost of the stealth blimp project.
3. Senator Verbosus's budget request is \$14M greater than the cost of the solar-powered cargo plane.

**Senators:** Loquacius, Repetitius, Tedi-  
us, Verbosus

**Projects:** Stealth blimp, Supersonic helicopter, Solar-powered cargo plane, Flying submarine

**Budgets:** \$58 million, \$72 million, \$79 million, \$86 million

	Blimp	Copter	Plane	Sub	\$58M	\$72M	\$79M	\$86M
Loquacius								
Repetitius								
Tedi- us								
Verbosus								
\$58M								
\$72M								
\$79M								
\$86M								

The file `caucus.pvs` contains a formalization of this problem in PVS. The solution is encoded as a set of lemmas. Use the PVS prover to verify this solution, that is, prove the lemmas.

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<sup>1</sup>Based on the logic puzzle “On and On,” *England’s Finest Logic Problems*, March 1996, p. 33.