Interpretation and Formalization of the Right-of-Way Rules

Victor A. Carreño[1[0000-0002-9338-4335]], Mariano M. Moscato[2[0000-0002-6468-9498]],

Paolo M. Masci[2[0000-0002-0667-7763]], and Aaron M. Dutle[3[0000-0002-8503-5514]]

[1] Compass Engineering, San Juan, PR, USA
victor.carreno@ymail.com
[2] National Institute of Aerospace, Hampton, VA, USA
{mariano.moscato, paolo.masci}@nianet.org
[3] NASA Langley Research Center, Hampton, VA, USA
aaron.m.dutle@nasa.gov

Abstract. This paper presents an interpretation and mathematical definition of the right-of-way rules as stated in USA, Title 14 of the Code of Federal Regulations, Part 91, Section 91.113 (14 CFR 91.113). In an encounter between two aircraft, the right-of-way rules defines which aircraft, if any, has the right-of-way and which aircraft must maneuver to stay well clear of the other aircraft.

The objective of the work presented in this paper is to give an unambiguous interpretation of the rules. From the interpretation, a precise mathematical formulation is created that can be used for analysis and proof of properties. The mathematical formulation has been defined in the Prototype Verification System (PVS) and properties of well formedness and core properties of the formalization have been mechanically proved. This mathematical formulation can be implemented digitally, so that right-of-way rules can be used in simulation or in future autonomous operations.

Keywords: right-of-way, safety, regulations.

1 Introduction

The right-of-way rules in 14 CFR 91.113 states that, weather conditions permitting, all operators of aircraft shall maintain vigilance so as to see and avoid other aircraft. This applies to both aircraft operating under instrument flight rules or visual flight rules. The rules contain operational considerations and physical considerations. Operational considerations include category of aircraft, phase of flight, and conditions such as aircraft in distress or refueling. Physical considerations embody the geometry of an encounter including position and velocity. The work presented in this paper focuses on the physical considerations of the rules.

There have been several previous efforts to characterize and mathematically formalize the right-of-way rules. In [1], the physical considerations of the rules are formalized, and safety and other properties are stated and proved. The rules are stated in [2] in a quantified way with considerations for what it means to be “well clear” of other aircraft, definitions of “ambiguous” and “unambiguous” scenarios, angles and geometries. The International Civil Aviation Organization, Rules of the Air document [3], define international right-of-way rules and additionally provides geometric definitions to some terms that are not defined in 14 CFR 91.113 such as “overtaking.” FAA, ATO (USA, Federal Aviation Administration, Air Traffic Organization) Order JO 7110.65 [4] provides definitions of terms regarding geometries and scenarios that might not be found in the regulations. In addition to the existing documents and technical papers, the authors have consulted with aviation experts in the interpretation of the rules [5][6].

The work presented in this paper shares some of the mathematical definitions in [1]. However, crucial concepts are redefined to capture a different interpretation of the rules. For example, in this paper, “convergence” is not defined in terms of closure rate but rather takes into consideration the location, geometries, and whether one aircraft has crossed the track of the other. The definition of “head-on, or nearly so” is extended to include difference in tracks that are 180 degrees, plus or minus an angle threshold. Also, the definitions consider the distance at which the aircraft will pass by projecting their trajectories. This is generally referred to as Horizontal Miss Distance (HMD) or projected distance at Closest Point of Approach (CPA). Even when the horizontal distance between the aircraft is decreasing, if the aircraft will pass at a sufficiently large distance, the scenario is not considered converging but rather “crossing.” This interpretation is consistent with aviation expert’s opinion [5][6]. A detailed comparison of the formalization in [1] and the formalization in this paper can be found in Section 4.

More generally, the work presented here can be seen as the formalization of an airspace operational concept, including the verification of properties that are intended to hold. Similar work in this vein includes the analysis of the Small Aircraft Transportation System [7], and the specification and analysis of parallel landing scenarios [8]. This work is also related to the interpretation, specification, and analysis of ambiguous natural language requirements. Interpreting written requirements into a precise formulation is a well-known problem [9], and purpose-built methods both manual [10] and automatic [11] are currently being used and refined for doing so. This work uses the Prototype Verification System (PVS) [12] to specify the interpretation of the right-of-way rules, and verify several core properties. This formal specification and verification is also envisioned to be used as a component in a rule-compliant pilot model being developed for simulation purposes.

2 Formalization of Right-of-Way Rules

Section 14 CFR 91.113 defines the following right-of-way rules:

(a) Inapplicability. (Not used in the Right-of-Way Rules formalization.)

(b) General. When weather conditions permit, regardless of whether an operation is conducted under instrument flight rules or visual flight rules, vigilance shall be maintained by each person operating an aircraft so as to see and avoid other aircraft. When a rule of this section gives another aircraft the right-of-way, the pilot shall give way to that aircraft and may not pass over, under, or ahead of it unless well clear.

(c) In distress. (Not used in the Right-of-Way Rules formalization.).

(d) Converging. When aircraft of the same category are converging at approximately the same altitude (except head-on, or nearly so), the aircraft to the other's right has the right-of-way.

(e) Approaching head-on. When aircraft are approaching each other head-on, or nearly so, each pilot of each aircraft shall alter course to the right.

(f) Overtaking. Each aircraft that is being overtaken has the right-of-way and each pilot of an overtaking aircraft shall alter course to the right to pass well clear.

The right-of-way rules, as defined in the regulations, deal with pairs of aircraft and do not consider cases where more than two aircraft are involved in an encounter. Also, the required actions in paragraphs (e) and (f) are specified in the horizontal domain and no alternatives are given for the vertical domain. The formalization presented in this paper does not attempt to "extend," "change," or "improve" on the regulations. The formalization attempts to be a faithful interpretation of the regulations.

In the formalization presented in this paper, the airspace is modeled by a 2-dimensional Cartesian flat-earth projections. Aircraft are assumed to be "at approximately the same altitude" and represented by position and velocity vectors. It should be noted that when aircraft are vertically separated (not at approximately the same altitude), they are considered to be well clear and there is no need for the right-of-way rules to be applied. Furthermore, aircraft that need to maneuver to remain well clear are horizontally close (~5 nautical miles) and so the flat-earth assumption incurs negligible errors.

Consider two-dimensional position vectors $\mathbf{s_0}, \mathbf{s_1}, \ldots, \mathbf{s_n} \in \mathbb{R}^2$ and two-dimensional non-zero (section 6 discusses this constraint) velocity vectors $\mathbf{v_0}, \mathbf{v_1}, \ldots, \mathbf{v_n} \in \mathbb{R}^2$. An aircraft A_0 is uniquely defined on the horizontal plane by its position and velocity vectors ($\mathbf{s_0}, \mathbf{v_0}$). For a vector $\mathbf{v_n} \in \mathbb{R}^2$, define $\mathbf{v_n}^\perp$ as the 90-degree clockwise rotation of $\mathbf{v_n}$ = ($v_{n,x}$, $v_{n,y}$) by $\mathbf{v_n}^\perp$ = ($v_{n,y}$, $-v_{n,x}$). The norm operator returning the magnitude of a vector $\mathbf{s_n}$ is represented by $\|\mathbf{s_n}\|$. For any two vectors $\mathbf{v_n}, \mathbf{v_m}$ the dot or scalar product is represented by $\mathbf{v_n} \cdot \mathbf{v_m}$.

2.1 Basic Definitions

The heading and position of an aircraft divide the horizontal plane into four quadrants as shown in Figure 1. The following definitions formalize this notion.

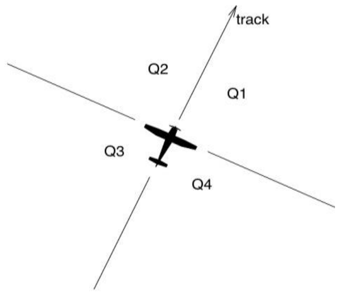


Fig. 1. Quadrants of the aircraft

Definition 1 (Quadrant). *Given an aircraft A_0 = (**s**$_0$, **v**$_0$), a position **s**$_1$ is in aircraft's A_0 first, second, third, or fourth quadrant (Q1, Q2, Q3, or Q4 respectively) when the corresponding of the following predicates hold:*

$$Q1(A_0, \boldsymbol{s_1}) := (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0^\perp} > 0 \ \wedge (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0} \geq 0 \tag{1}$$

$$Q2(A_0, \boldsymbol{s_1}) := (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0^\perp} \leq 0 \ \wedge (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0} > 0 \tag{2}$$

$$Q3(A_0, \boldsymbol{s_1}) := (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0^\perp} < 0 \ \wedge (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0} \leq 0 \tag{3}$$

$$Q4(A_0, \boldsymbol{s_1}) := (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0^\perp} \geq 0 \ \wedge (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0} < 0 \tag{4}$$

Definition 2 (Track). The track of an aircraft A_0 = (**s**$_0$, **v**$_0$) is defined as the angle between the north and the direction of an aircraft, measured in a clockwise direction. For example, an aircraft flying west in on a 270-degree track.

$$trk(A_0) := \tan^{-1}\left(\frac{v_{0,x}}{v_{0,y}}\right) \tag{5}$$

A trajectory is defined as the linear projection of the aircraft's velocity vector. The Closest Point of Approach (CPA) is the geometrical condition when the trajectories of two aircraft will be at the smallest distance or range. The time to CPA is the time from the current position to the moment when the distance is smallest when their velocity vectors are projected in a straight trajectory. The time is positive when the CPA will occur in the future and negative when the CPA has already occurred. If the velocity vectors are parallel in the same direction and their magnitudes are equal, the current distance between aircraft will not change and is the CPA. In this case, the time to CPA is defined to be zero.

Definition 3 (Time to Closest Point of Approach). *For aircraft A_0 = (**s**$_0$, **v**$_0$) and A_1 = (**s**$_1$, **v**$_1$) the time to closest point of approach is defined as:*

$$t_{CPA}(A_0, A_1) := \begin{cases} -\frac{(s_0 - s_1)\cdot(v_0 - v_1)}{(v_0 - v_1)\cdot(v_0 - v_1)} & if\ \boldsymbol{v_0} \neq \boldsymbol{v_1} \\ 0 & if\ \boldsymbol{v_0} = \boldsymbol{v_1} \end{cases} \tag{6}$$

Definition 4 (Horizontal Miss Distance). *The horizonal miss distance of two aircraft A_0 = ($\mathbf{s_0}$, $\mathbf{v_0}$) and A_1 = ($\mathbf{s_1}$, $\mathbf{v_1}$) is the distance at the Closest Point of Approach when their position and velocity vectors are projected in time:*

$$HMD(A_0, A_1) := \|(\boldsymbol{s_0} - \boldsymbol{s_1}) + t_{CPA}(A_0, A_1)(\boldsymbol{v_0} - \boldsymbol{v_1})\| \quad (7)$$

The trajectories of two aircraft on a two-dimensional airspace with non-zero velocity vectors are either going to cross (in the future) or already crossed (in the past) unless their velocity vectors are parallel. Figure 2 illustrates the three cases of a generic crossing. In the first case, neither aircraft has passed the crossing point. In the second, only A_0 has crossed the trajectory of A_1. In the third, each aircraft has crossed the other's trajectory.

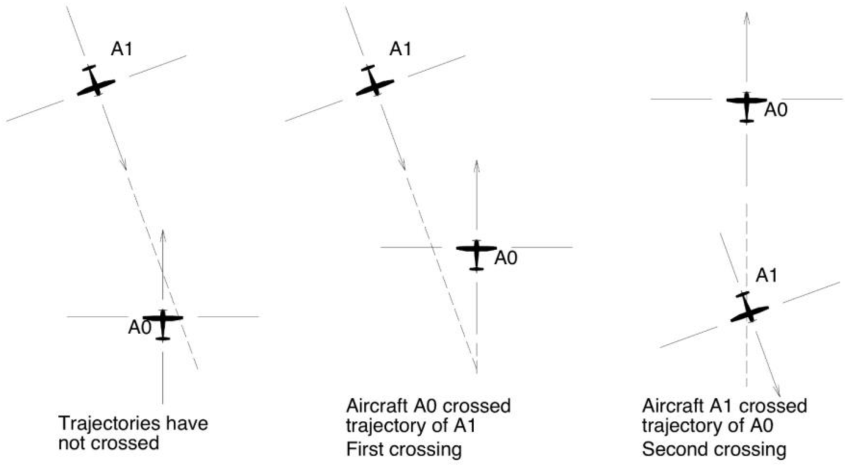


Fig. 2. Before and after trajectory crossing

In order to define the crossing of aircraft, the relative position and the relative motion of an aircraft with respect to another are defined.

Definition 5 (Relative Position). *A position* $\mathbf{s_1}$ *is to the left of (respectively to the right of) an aircraft A_0 = ($\mathbf{s_0}$, $\mathbf{v_0}$) when the following predicates hold (respectively):*

$$to\_the\_left\_of(A_0, \boldsymbol{s_1}) := (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0}^{\perp} < 0 \quad (8)$$

$$to\_the\_right\_of(A_0, \boldsymbol{s_1}) := (\boldsymbol{s_1} - \boldsymbol{s_0}) \cdot \boldsymbol{v_0}^{\perp} > 0 \quad (9)$$

Definition 6 (Relative Motion). *An aircraft A_1 = ($\mathbf{s_1}$, $\mathbf{v_1}$) is moving left to right (respectively right to left) with respect to aircraft A_0 = ($\mathbf{s_0}$, $\mathbf{v_0}$) when the following predicates hold (respectively):*

$$left\_to\_right(A_0, A_1) := v_0 \cdot v_1^{\perp} < 0 \tag{10}$$

$$right\_to\_left(A_0, A_1) := v_0 \cdot v_1^{\perp} > 0 \tag{11}$$

Using the definitions above, it is possible to formalize situational notions describing scenarios in which aircraft have or have not crossed each other's trajectories.

Definition 7 (Going to Cross). *An aircraft A_1 = ($\mathbf{s_1}$, $\mathbf{v_1}$) is going to cross the trajectory of A_0 = ($\mathbf{s_0}$, $\mathbf{v_0}$) (in the future) if either it is to the left of A_0 and it is moving left to right of if it is to the right of A_0 and it is moving right to left:*

$$\begin{aligned} going\_to\_cross(A_0, A_1) := \quad & (to\_the\_left\_of(A_0, \boldsymbol{s_1}) \wedge left\_to\_right(A_0, A_1)) \vee \\ & (to\_the\_right\_of(A_0, \boldsymbol{s_1}) \wedge right\_to\_left(A_0, A_1)) \end{aligned} \tag{12}$$

Definition 8 (Already Crossed). *Aircraft A_1 already crossed the trajectory of A_0 (in the past) if it is to the left of A_0 and it is moving right to left or if it is to the right of A_0 and it is moving left to right:*

$$\begin{aligned} crossed(A_0, A_1) := \quad & (to\_the\_left\_of(A_0, \boldsymbol{s_1}) \wedge right\_to\_left(A_0, A_1)) \vee \\ & (to\_the\_right\_of(A_0, \boldsymbol{s_1}) \wedge left\_to\_right(A_0, A_1)) \end{aligned} \tag{13}$$

Definition 9 (Zero Crossed). *The trajectories of two aircraft are going to cross (in the future) when neither aircraft have crossed the trajectory of the other:*

$$zero\_crossed(A_0, A_1) := going\_to\_cross(A_0, A_1) \wedge going\_to\_cross(A_1, A_0) \tag{14}$$

Definition 10 (One crossed). *One aircraft has crossed the trajectory of the second but the second has not crossed the trajectory of the first:*

$$\begin{aligned} one\_crossed(A_0, A_1) := \quad & (going\_to\_cross(A_0, A_1) \wedge crossed(A_1, A_0)) \vee \\ & (going\_to\_cross(A_1, A_0) \wedge crossed(A_0, A_1)) \end{aligned} \tag{15}$$

Definition 11 (Both crossed). *Both aircraft have crossed each other's trajectories:*

$$both\_crossed(A_0, A_1) := crossed(A_0, A_1) \wedge crossed(A_1, A_0) \tag{16}$$

Definition 12 (Parallel). *The trajectories of aircraft A_0 and A_1 are parallel when the following predicate holds:*

$$parallel(A_0, A_1) := \boldsymbol{v_0} \cdot \boldsymbol{v_1^{\perp}} = 0 \tag{17}$$

Definition 13 (Same Orientation). *The following predicate holds when the angle between the trajectories of aircraft A_0 and A_1 is acute:*

$$same\_orientation(A_0, A_1) := v_0 \cdot v_1 > 0 \tag{18}$$

Definition 14 (Opposite Orientation). *The following predicate holds when the angle between the trajectories of aircraft A0 and A1 are obtuse:*

$$opposite\_orientation(A_0, A_1) := v_0 \cdot v_1 < 0 \tag{19}$$

Definition 15 (In Q1 and was in Q2). *An aircraft A_1 is in the first quadrant of aircraft A_0 and was in the second quadrant of aircraft A_0 when the following predicate holds:*

$$is\_in\_q1\_and\_was\_in\_q2(A_0, A_1) := \begin{array}{l}(Q1(A_0, \boldsymbol{s_1}) \wedge (\, Q4(A_1, \boldsymbol{s_0}))\vee \\ (opposite\_orientation(A_0, A_1) \wedge \\ \quad left\_to\_right(A_0, A_1)))\end{array} \tag{20}$$

2.2 Convergence, Divergence, and Overtake

The concept of convergence, head-on or nearly so, and overtaking used in the right-of-way rules can be characterized by the relative quadrant location of each aircraft with respect to the other, the track angle between them, and their horizontal miss distance. Figure 3 shows examples of relative quadrant locations: aircraft A_1 in aircraft A_0's quadrant Q1 and aircraft A_0 in aircraft A_1's quadrants Q1 to Q4.

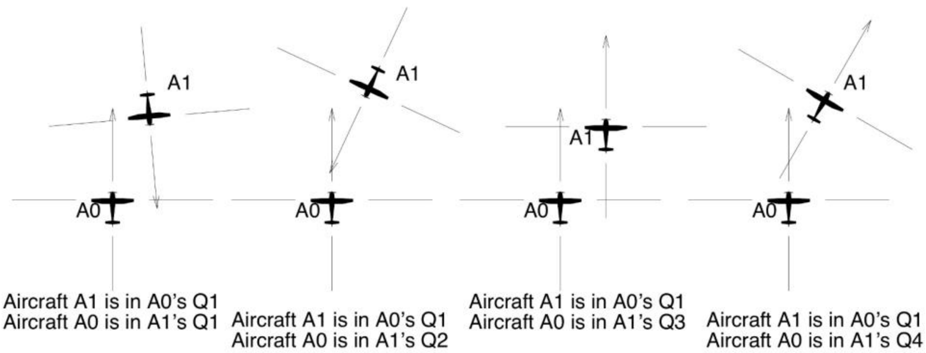


Fig. 3. Aircraft A_0 in aircraft A_1's quadrants Q1 to Q4

Table 1 shows the quadrant locations that are required for aircraft A_0 and A_1 to be converging, diverging, or overtaking. The combination of quadrants is necessary but not sufficient for convergence and overtake. For aircraft to be converging or overtaking, an HMD less than some threshold must also be satisfied. The HMD threshold is important because it makes an operational distinction between aircraft that are converging and aircraft whose trajectories are crossing but are not considered converging. For example, the trajectory of an aircraft over the Mediterranean Sea might be crossing the trajectory of an aircraft over Australia. However, these two aircraft are not operationally converging. The formalization leaves HMD as a parameter to be defined depending on the type of operation.

Table 1. Quadrant location for convergence and overtake requirement

A_0/A_1	Q1	Q2	Q3	Q4
Q1	Convergence	Convergence	Overtake	Overtake
Q2	Convergence	Convergence	Overtake	Overtake
Q3	Overtake	Overtake	Divergence	Divergence
Q4	Overtake	Overtake	Divergence	Divergence

Definition 16 (General convergence). *Let $A_0 = (\mathbf{s_0}, \mathbf{v_0})$ and $A_1 = (\mathbf{s_1}, \mathbf{v_1})$ be aircraft and δ_C a nonnegative real number. A_0 and A_1 are converging when the following predicate holds:*

$$converging(A_0, A_1)(\delta_C) := Q_C(A_0, A_1) \wedge (HMD(A_0, A_1) < \delta_C) \tag{21}$$

where the predicate $Q_C(A_0,A_1)$ in the equation above are the quadrant locations of A_0 with respect to A_1 and the location of A_1 with respect to A_0:

$$\begin{aligned} Q_C(A_0, A_1) := \quad & (Q1(A_0, \boldsymbol{s_1}) \wedge Q1(A_1, \boldsymbol{s_0})) \vee (Q1(A_0, \boldsymbol{s_1}) \wedge Q2(A_1, \boldsymbol{s_0})) \vee \\ & (Q2(A_0, \boldsymbol{s_1}) \wedge Q1(A_1, \boldsymbol{s_0})) \vee (Q2(A_0, \boldsymbol{s_1}) \wedge Q2(A_1, \boldsymbol{s_0})) \end{aligned} \tag{22}$$

The general convergence definition above includes cases where the aircraft could be head-on or nearly so. An additional constraint is put in place to exclude the head-on or nearly so cases.

Definition 17 (Convergence, not head-on). *Aircraft A_0 and A_1 are converging but not head-on when they are converging and the angle between the aircraft tracks is greater than180 degrees plus θ_H or less than 180 degrees minus θ_H, where θ_H is the angular threshold:*

$$\begin{aligned} conv\_not\_headon(A_0, A_1)(\delta_C, \theta_H) := \quad & Q_C(A_0, A_1) \wedge (HMD(A_0, A_1) < \delta_C) \wedge \\ & (180 + \theta_H < |trk(A_0) - trk(A_1)| \vee \\ & |trk(A_0) - trk(A_1)| < 180 - \theta_H) \end{aligned} \tag{23}$$

Definition 18 (Head-on, or nearly so). *Aircraft A_0 and A_1 are head-on, or nearly so, when they are converging and the difference in their tracks is 180 degrees plus or minus θ_H:*

$$headon(A_0, A_1)(\delta_C, \theta_H) := \begin{array}{c} Q_C(A_0, A_1) \land (HMD(A_0, A_1) < \delta_C) \land \\ (180 - \theta_H \leq |trk(A_0) - trk(A_1)| \leq 180 + \theta_H) \end{array} \quad (24)$$

Definition 19 (Overtaking). *Aircraft A_0 is overtaking aircraft A_1 when aircraft A_1 is in quadrants Q1 or Q2 of aircraft A_0 and aircraft A_0 is in quadrants Q3 and Q4 of aircraft A_1 and the HMD is less than a threshold:*

$$overtaking(A_0, A_1)(\delta_O) := \begin{array}{c} (Q1(A_0, \boldsymbol{s_1}) \lor Q2(A_0, \boldsymbol{s_1})) \land \\ (Q3(A_1, \boldsymbol{s_0}) \lor Q4(A_1, \boldsymbol{s_0})) \land \\ (HMD(A_0, A_1) < \delta_O) \end{array} \quad (25)$$

Definition 20 (Right-of-way). *Aircraft A_1 = ($\boldsymbol{s_1}$, $\boldsymbol{v_1}$) has the right-of-way and aircraft A_0 = ($\boldsymbol{s_0}$, $\boldsymbol{v_0}$) has to give way to A_1 when A_0 is overtaking A_1 or the aircraft are converging (except head-on or nearly so), A_1 is to the right of A_0, and their trajectories have not crossed:*

$$right\_of\_way(A_1, A_0)(\delta_O, \delta_C, \theta_H) := \begin{array}{c} overtaking(A_0, A_1)(\delta_O) \lor \\ (conv\_not\_headon(A_0, A_1)(\delta_C, \theta_H) \land \\ to\_the\_right\_of(A_0, \boldsymbol{s_1}) \land \\ zero\_crossed(A_0, A_1)) \end{array} \quad (26)$$

3 Properties of the Right-of-Way Rules

This section presents properties of the right-of-way rules. These properties ensure that the right-of-way rules, as interpreted above, do not conflict with the sense of the rules. They were each specified and proved in the Prototype Verification System (PVS).

Theorem 1 (Safety 1). *For all aircraft A_0 and A_1 and all $\delta_O, \delta_C, \theta_H \in \mathbb{R}_{\geq 0}$ it is never the case that both aircraft have the right-of-way at the same time.*

$$\begin{array}{c} right\_of\_way(A_1, A_0)(\delta_O, \delta_C, \theta_H) \Rightarrow \\ \neg right\_of\_way(A_0, A_1)(\delta_O, \delta_C, \theta_H) \end{array}$$

Theorem 2 (Safety 1a). *For all aircraft A_0 and A_1, if the aircraft have not crossed trajectories, then one and only one aircraft is to the right of the other.*

$$\begin{array}{c} \mathrm{zero\_crossed}(A_1, A_0) \Rightarrow \\ (to\_the\_right\_of(A_0, \boldsymbol{s_1}) \land \neg to\_the\_right\_of(A_1, \boldsymbol{s_0})) \lor \\ (to\_the\_right\_of(A_1, \boldsymbol{s_0}) \land \neg to\_the\_right\_of(A_0, \boldsymbol{s_1})) \end{array}$$

Theorem 3 (Overtaking is Asymmetric). *For all aircraft A_0 and A_1 and all $\delta_O \in \mathbb{R}_{\geq 0}$, if A_0 is overtaking A_1 then A_1 is not overtaking A_0.*

$$overtaking(A_0, A_1)(\delta_O) \Rightarrow \neg overtaking(A_1, A_0)(\delta_O)$$

Theorem 4 (No right-of-way after crossing). *For all aircraft A_0 and A_1, after the first aircraft has crossed the trajectory of the second, but before the second has crossed the trajectory of the first, both of them will be to the right of each other or both will be to the left of each other.*

$$\begin{gathered} one\_crossed(A_0, A_1) \Rightarrow \\ (to\_the\_right\_of(A_0, \boldsymbol{s_1}) \wedge to\_the\_right\_of(A_1, \boldsymbol{s_0})) \vee \\ (to\_the\_left\_of(A_0, \boldsymbol{s_1}) \wedge to\_the\_left\_of(A_1, \boldsymbol{s_0})) \end{gathered}$$

This is a geometrical property and not a right-of-way rules property. However, it has a right-of-way rules implication. If both aircraft are to the right of each other, then according to rule (d), they both have right-of-way. However, according to Theorem 1, both cannot have right-of-way simultaneously. If both are to the left of each other, then rule (d) does not apply. The implication is that when one aircraft has crossed the trajectory of the other, neither aircraft has right-of-way.

Theorem 5 (Awareness). *When any two aircraft A_0, A_1 are converging (not head-on or nearly so) it is sufficient for an aircraft to know if one of the aircraft has crossed the trajectory of the other and if the traffic is to its right to determine if it has right-of-way.*

$$\begin{gathered} conv\_not\_headon(A_0, A_1)(\delta_C, \theta_H) \Rightarrow \\ (zero\_crossed(A_1, A_0) \wedge to\_the\_right\_of(A_0, \boldsymbol{s_1}) \\ \Rightarrow right\_of\_way(A_1, A_0)(\delta_O, \delta_C, \theta_H)) \end{gathered}$$

Theorem 6 (Had right-of-way). *For any two aircraft A_0, A_1, to be in each other's first quadrant, at least one had to cross from the other's second quadrant.*

$$\begin{gathered} \mathrm{Q1}(A_0, \boldsymbol{s_1}) \wedge \mathrm{Q1}(A_1, \boldsymbol{s_0}) \wedge \neg parallel(A_0, A_1) \Rightarrow \\ (is\_in\_q1\_and\_was\_in\_q2(A_0, A_1) \vee is\_in\_q1\_and\_was\_in\_q2(A_1, A_0)) \end{gathered}$$

The significance of this theorem is that if the aircraft are in each other's first quadrant, and neither have right of way because both are to the right of each other, then before the crossing, one of them was to the right and one was to the left and hence one had right of way and the other did not.

4 Comparison with Previous Formalization

This section presents a detailed account of the differences between the definitions presented in [1] and the interpretation and definitions in this work.

4.1 Converging

The definition of convergence in [1] is based on closure rate. If the horizontal distance between the aircraft is getting smaller, the aircraft are converging. The implication of this definition is that given the speeds of the aircraft are approximately equal, the following two cases in Figure 4 are considered horizontally converging:

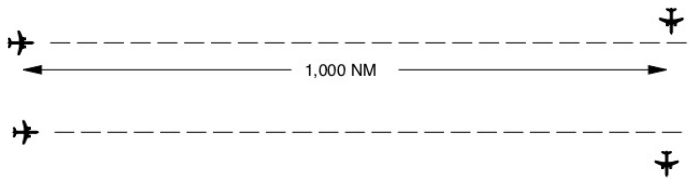


Fig. 4. Cases 1 and 2, cases considered converging as defined in [1]

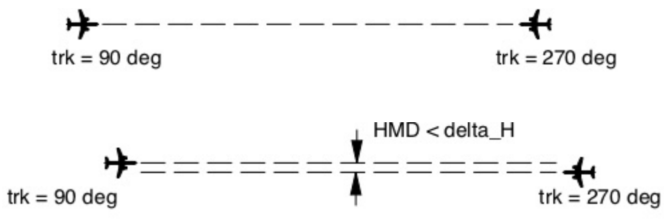
To operational personnel, such as Air Traffic Controllers, these two cases are not converging scenarios, the first because the horizontal miss distance is large, and the second because the aircraft that has crossed the trajectory of the other is leaving the other aircraft behind.

In this paper, convergence is defined by the relative location of one aircraft with respect to the location and motion of the other and the Horizontal Miss Distance (HMD). Case 1 does not satisfy the definition of convergence because $HMD > \delta_C$ (assuming δ_C is significantly smaller that a distance in the order of 1,000 nautical miles) and Case 2 does not satisfy the definition of convergence because the aircraft to the left is in Quadrant 4 of the aircraft to the right and $Q_C(A_0, A_1)$ is false.

4.2 Head-on or nearly so

Paragraph (e) of 14 CFR 91.113 alludes to the scenario as "head-on or nearly so." Narckawicz et. al. [1] defines head-on as when the aircraft have trajectories strictly 180 degrees opposite each other and when their trajectories are perfectly aligned with the line segment connecting their positions. This definition does not include the "or nearly so" part of the definition in the regulations.

In Figure 5 below, Case 3 satisfy the definition of head-on in [1]. However, Case 4 and Case 5, where the aircraft are head-on or nearly so, do not satisfy the definition in [1].



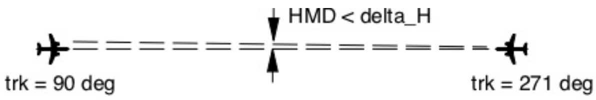


Fig. 5. Cases 3 to 5, cases illustrating strictly head-on and head-on or nearly so

In this paper, head-on or nearly so is defined by specifying the relative quadrant with respect to each other, a Horizontal Miss Distance and a range of angles. This definition is satisfied by cases 3, 4, and 5 above (assuming θ_H is greater or equal to one degree).

4.3 Overtaking

The definition of overtaking in [1] suffers from the same shortcoming as the definition of head-on. Only geometries where the tracks are perfectly aligned and the trajectory difference is zero are considered to be overtaking. Scenarios where the trajectories are displaced sideways by any distance greater than zero or the difference in trajectory angle is not zero do not satisfy the definition of overtaking.

This paper defines overtaking in terms of the relative quadrants and the Horizontal Miss Distance. Scenarios where the difference in track angles is greater than zero and the tracks are not perfectly aligned satisfy this definition, as long as the HMD is less than the given threshold.

4.4 Right-of-way

The main objective of the right-of-way rules is to determine, based on geometry and state, which aircraft, in an encounter between a pair of aircraft, has the right-of-way, if any. There are major differences in the definitions of right-of-way in [1] and in this paper. These differences lead to different aircraft having right-of-way in the same scenario, as it will be shown below.

There are four shortcomings with the right-of-way definition in [1]:

First, in scenarios where the aircraft are nearly head-on, this definition would give one of the aircraft the right-of-way when, in reality, neither aircraft should have right-of-way and both aircraft should turn right to stay well clear as stated in paragraph (e) of 14 CFR 91.113. Case 8 in Figure 6 shows two aircraft, A1 and A2, in a nearly head-on encounter scenario. The right-of-way definition in [1] improperly gives aircraft A2 the right-of-way.

Fig. 6. Case 8, Nearly head-on

Second, when two aircraft are converging and neither have crossed the other's trajectory, paragraph (d) of 14 CFR 91.113 gives one of them the right-of-way. After one crosses the trajectory of the other, they both will be to the right of each other or both will be to the left. However, the definition in [1] continues to give the right-of-way to the aircraft that was to the right before the crossing. In some scenarios, this could be problematic. Case 9 in Figure 7 shows a converging crossing scenario where A2 is to the right of A1 and has the right-of-way. After A1 crosses the trajectory of A2, A2 will be mostly or completely out of the field of view of A1. At this point, A2 should not continue to have right-of-way. This state will be more applicable to an overtaking situation where the aircraft behind will have to give way to the aircraft in front.

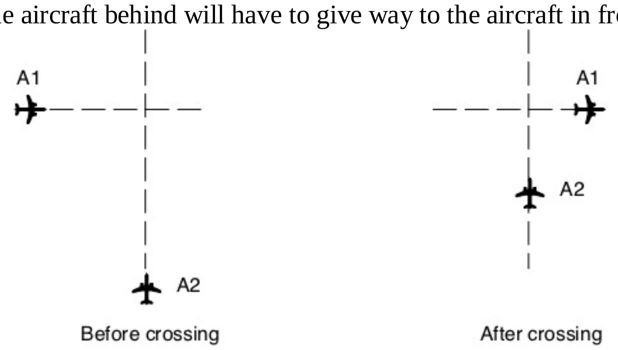


Fig. 7. Case 9, before and after crossing and persistence of right-of-way

Third, the right-of-way definition in [1] gives right-of-way to an aircraft in scenarios where the aircraft are diverging. This is not covered in 14 CFR 91.113 regulations. That is, for diverging aircraft, the regulations do not give right-of-way to either aircraft.

Finally, in scenarios where an aircraft is overtaking another, but the difference in their trajectory angles is not zero and/or their trajectories are not perfectly aligned, the definition of right-of-way in [1] improperly gives the right-of-way to the aircraft that is to the right and not to the aircraft that is being overtaken. Case 10 in Figure 8 shows a scenario where aircraft A1 is overtaking A2. However, according to the definition in [1], A1 has right of way instead of A2, which contradicts paragraph (f) of 14 CFR 91.113. The issue arises from the definition of overtaking in [1], which does not consider Case 10 of Figure 8 as an overtaking scenario.

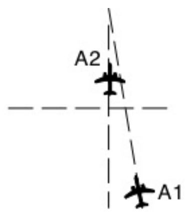


Fig. 8. Case 10, Overtaking

The right-of-way definition in this paper gives right-of-way to the aircraft that is being overtaken, but the definition of overtaking is not restricted to same trajectory, zero angle cases. It also gives right-of-way to the aircraft that is to the right in a converging scenario, but it excludes head-on or nearly so, and limits the definition of convergence to scenarios where aircraft are on quadrants Q1 or Q2 of each other.

For the right-of-way definition in this paper, aircraft A1 has right-of-way when A0 is overtaking A1 or when the aircraft are converging, are not head-on or nearly so, A1 is to the right of A0, and their trajectories have not crossed.

The authors believe that this is the correct definition of the right-of-way rules described in 14 CFR 91.113 and the interpretation of these regulations.

5 Aspects of the Right-of-Way Rules Formalization

This section discusses aspects of the right-of-way rules, the definitions, and formalization. In parts of the right-of-way rules, the following terms are used: “may not pass over, under, or ahead”, “when aircraft of the same category are converging”, “when aircraft are approaching each other”, and “each aircraft that is being overtaken.”

The rules are defined in terms of motion, either in relative terms or in absolute terms. In order for two aircraft to have relative motion, at least one of them has to be moving with respect to the other or with respect to a reference. Hence, when formalizing the right-of-way rules in mathematical terms, the constraint is put in place that there is relative motion between the aircraft. That is, that the velocity vector of at least one aircraft is non-zero. If there is no relative motion between the aircraft, it will be impossible to pass, to converge, to approach, or to overtake.

Other considerations in the formalization of the right-of-way rules is the notion of direction, location of the aircraft respect to the other (quadrant), head-on, and right or left. In general, an aircraft flies in the direction of its longitudinal axis. However, there are many instances when the motion of an aircraft is not aligned with its longitudinal axis. For example, a rotorcraft could be moving perpendicular to its longitudinal axis. A slow aircraft in the presence of strong winds could be moving, relative to ground, along its longitudinal axis but opposite of what is consider the front of the aircraft (moving backward with respect to the ground). An aircraft in the presence of a crossed wind will not have its longitudinal axis aligned with its direction of travel.

There are some instances that are not covered by the right-of-way rules. An encounter where an aircraft is stationary (zero velocity vector) is not covered. For example, when an aircraft A1 is approaching a stationary aircraft A2, it is not possible to determine whether the aircraft are converging head-on or being overtaken. It is assumed that the intent of the rules is that paragraph (b) applies and that “vigilance shall be maintained by each person operating an aircraft so as to see and avoid other aircraft” but that neither paragraph (e) nor (f) applies and that neither of the aircraft has right of way.

6 Summary and Conclusion

The paper presents an interpretation and a mathematical definition of the right-of-way rules as defined in the US Title 14 Code of Federal Regulations 91.113. The main contributions are: (i) a formalization of the regulations that align with the exact definition of the regulations and with the interpretation by operational experts; (ii) a mechanized analysis in the Prototype Verification System (PVS) of well formedness and core properties of the formalization; (iii) a detailed discussion of the differences between the formalization presented in this paper and the one found in [1]. Additional objectives are to use the mathematical formulation presented in this paper to develop a rule compliant virtual pilot that can be used in simulation experiments and possibly be used in future autonomous vehicles. Planned future work includes the formulation of stability properties of the right-of-way rules and the mechanized proof of these properties.

Acknowledgements.

Funding for the first author has been provided by the National Aeronautics and Space Administration. Research by the second and third authors is supported by the National Aeronautics and Space Administration under NASA/National Institute of Aerospace Cooperative Agreement NNL09AA00A.

References

1. Narkawicz, Anthony; Muñoz, Cesar A., Maddalon, Jeffrey: "A Mathematical Analysis of Air Traffic Priority Rules," Proceedings of the 12th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference, AIAA-2012-5544, Indianapolis, Indiana, 2012.
2. RTCA DO-365A: "Minimum Operational Performance Standards (MOPS) for Detect and Avoid (DAA) Systems," Appendix H, February 2020.
3. International Civil Aviation Organization: Rules of the Air, Annex 2 to the Convention on International Civil Aviation, Tenth Edition, July 2005.
4. U.S. Department of Transportation, Federal Aviation Administration, Air Traffic Organization, Order JO 7110.65X, October 2017.
5. Arbuckle, Douglas: Personal communications regarding interpretation of convergence and divergence, March 2021.
6. Minck, Jeffrey: Personal communications regarding interpretation of convergence and divergence, March 2021.
7. César Muñoz, Víctor Carreño, and Gilles Dowek: "Formal Analysis of the Operational Concept for the Small Aircraft Transportation System," Rigorous Engineering of Fault-Tolerant Systems, Lecture Notes in Computer Science, Vol. 4157, pp 306-325, 2006, doi 10.1007/11916246_16.
8. V. Carreño and C. Muñoz: "Formal Analysis of Parallel Landing Scenarios," 19th DASC. 19th Digital Avionics Systems Conference. Proceedings (Cat. No.00CH37126), 2000, pp. 1D6/1-1D6/6 vol.1, doi 10.1109/DASC.2000.886893.
9. Unnati S. Shah and Devesh C. Jinwala: "Resolving Ambiguities in Natural Language Software Requirements: A Comprehensive Survey," ACM SIGSOFT Software Engineering Notes 40, no. 5, pp. 1-7, (2015), doi 10.1145/2815021.2815032.
10. Mavridou, A., Bourbouh, H., Giannakopoulow, D., Pressburger, T., Hejase, M., Garoche, P., and Schummann, J. (2020). The Ten Lockheed Martin Cyber-Physical Challenges: Formalized, Analyzed, and Explained. 2020 IEEE 28th International Requirements Engineering Conference (RE), 300-310.

11. Shalini Ghosh, Daniel Elenius, Wenchao Li, Patrick Lincoln, Natarajan Shankar, and Wilfrid Steiner. "ARSENAL: Automatic Requirements Specification Extraction from Natural Language." In NASA Formal Methods Symposium, pp. 41-46. Springer, Cham, 2016.
12. Owre, S., Rushby, J.M., Shankar, N.: PVS: A Prototype Verification System. In: Automated Deduction – CADE-11, 11th International Conference on Automated Deduction. Lecture Notes in Computer Science, vol. 607, pp 748-752. Springer (1992), https://doi.org/10.1007/3-540-55602-8_217.