ECE 741/841

5 September 2002

Predicate Logic - First Order Logic

In propositional logic, we were able to represent sentences by propositional atoms. We were able to combine these atoms using connectives to define relations between sentences. However, we had difficulty expressing more subtle details about sentences.

"My sister wants a black and white cat."

could be encoded by a single propositional atom. However, it was difficult to express ideas about cats and the color of cats.

Predicate logic is a richer and more expressive language (and more complex) which allow us to express more accurately the concepts we want to describe.

Predicates

In english, (and other natural languages) the predicate is the part of a sentence that says something about the noun.

$$\text{Jane } \underbrace{\text{is a basketball player.}}_{\text{predicate}}$$

noun

In logic, predicates also say something about an object.

If we define CAT to be a predicate over objects of type animal, then

CAT(a) will be true if a is a cat and false if it is not.

Example

Let's define predicates and nouns to express

"My sister wants a black and white cat."

MS(x) - x is my syster
CAT(x) - x is a cat
BW(x) - x is black and white
W(x,y) - x wants y

$MS(z) \rightarrow W(z,w) \wedge CAT(w) \wedge BW(w)$

interpretation of implication

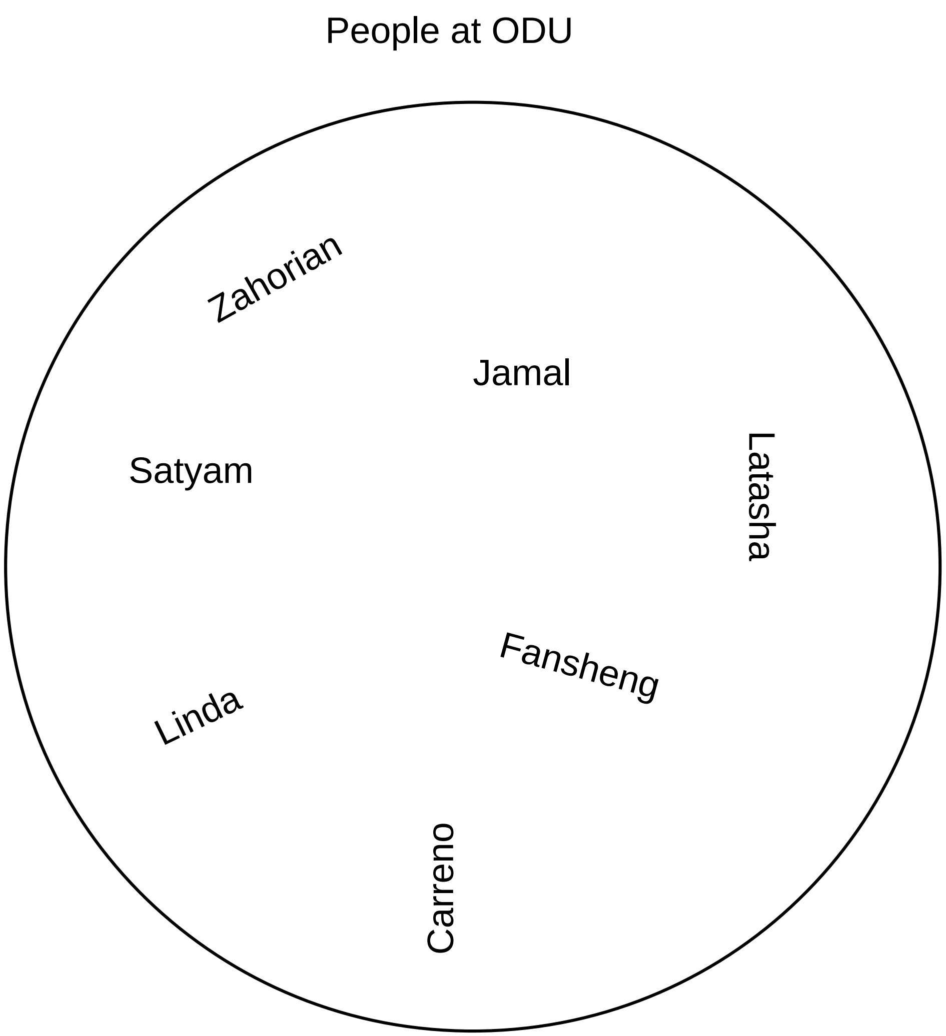
Predicates

Note that predicates can say something about an object, or express a relation between two or more objects.

BP(x) - x is a basketball player - says something about an object.
SYB(x,y,z) - x,y,z are syblings - defines a relation between objects.

In general, predicates can be unary, binary, ... , n-ary

Domains


People at ODU
Zahorian
Jamal
Satyam
Latasha
Fansheng
Linda
Carreno

Domains

The property *is a student* is either true or false of the elements in the domain.

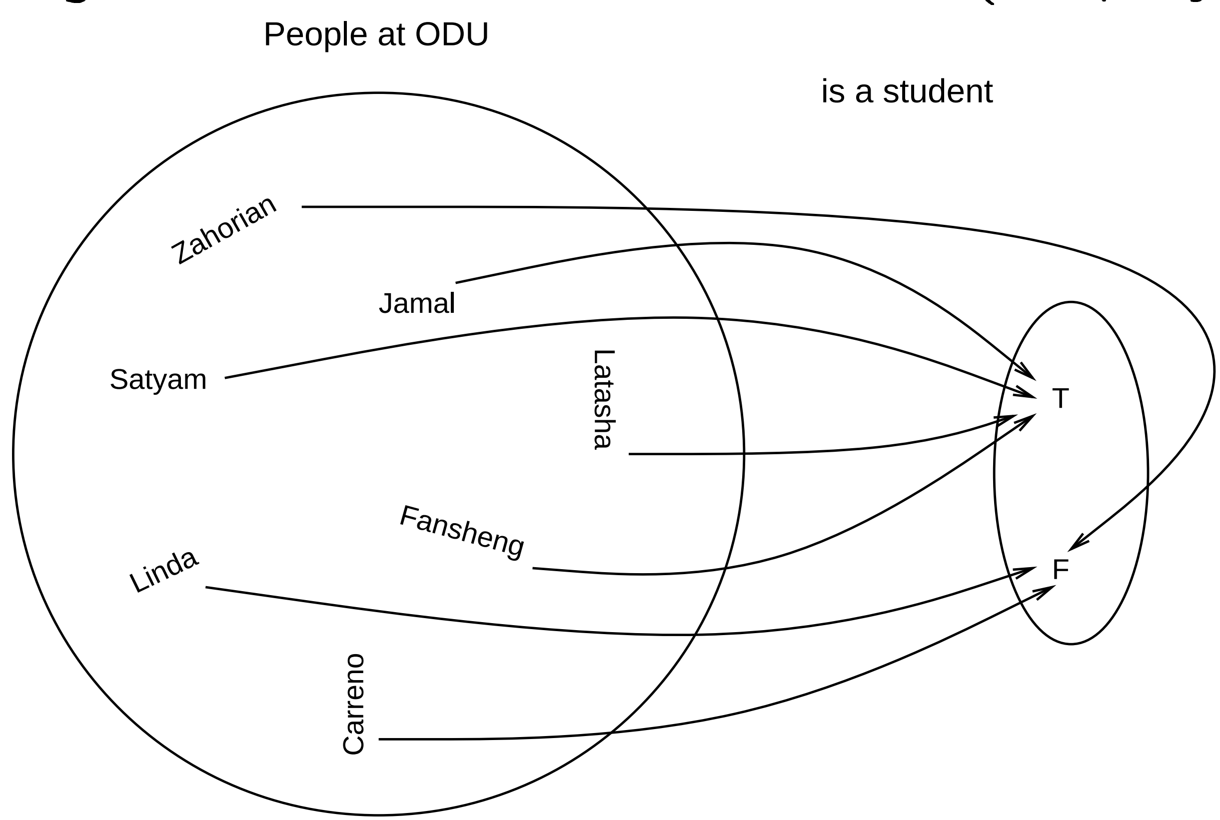
S(x) - x is a student

When we define a predicate, we also define the domain of the predicate. If the domain is *people at ODU*, then we should not apply the predicate to any person, nor should we apply another predicate defined over a different domain to the *people at ODU* domain.

The predicate CAT is defined over animals and should not have a *people at ODU* argument.

Predicates

Predicates are a special kind of function; they take an argument and return either T or F (uniquely)



Quantification

How can I express a sentence such as,

There are some students at ODU that are shorter than 1.7 meters.

All students at ODU are clever.

Predicate logic uses quantification to express the condition on which a predicate ranges over a domain.

let x be a variable that ranges over students at ODU and let SHO1.7(x) - x is shorter than 1.7 meters.

$\exists x \quad SHO1.7(x)$

There exists students that are shorter than 1.7 meters.

Quantification, cont

let x be a variable that ranges over students at ODU and let CL(x) - x is clever.

$\forall x \quad CL(x)$

For all students at ODU, they are clever.

Example

Every person is younger than his/her mother.

Example

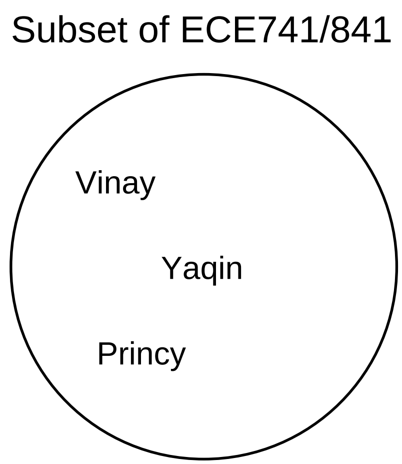
Domain - every person in the world

Y(x,y) - x is younger than y

M(x,y) - x is the mother of y

$\forall w \; \forall z \quad M(w,z) \rightarrow Y(z,w)$

The Meaning of Quantifiers



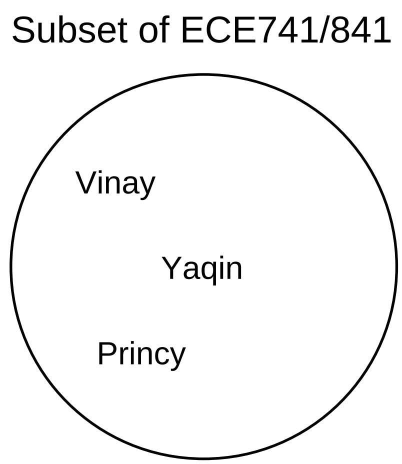
There are some students in the *subset of ECE 741/841* that are from India.

I(x) - x is from india.

$\exists x \quad I(x)$

$I(Vinay) \vee I(Yaqin) \vee I(Princy)$

The Meaning of Quantifiers, cont



All of the students in the *subset of ECE 741/841* are clever.

CL(x) - x is clever.

$\forall x \quad CL(x)$

$CL(Vinay) \wedge CL(Yaqin) \wedge CL(Princy)$

Example

Every student is younger than some instructor.

Example

let the domain be students and faculty at ODU.

S(x) - x is a student.
I(x) - x is an instructor.
Y(x,y) - x is younger than y.

$\forall x \; S(x) \rightarrow \exists y \; I(y) \wedge Y(x, y)$

$$(S(Mukund) \rightarrow \exists y \quad I(y) \wedge Y(Mukund, y)) \wedge$$
$$(S(Emre) \rightarrow \exists y \quad I(y) \wedge Y(Emre, y)) \wedge$$
$$(S(Gonzalez) \rightarrow \exists y \quad I(y) \wedge Y(Gonzalez, y)) \wedge$$
$$(S(Arturo) \rightarrow \exists y \quad I(y) \wedge Y(Arturo, y))...$$

Example

$$(S(Mukund) \rightarrow \exists y \quad I(y) \wedge Y(Mukund, y)) \wedge$$
$$(S(Emre) \rightarrow \exists y \quad I(y) \wedge Y(Emre, y)) \wedge$$
$$(S(Gonzalez) \rightarrow \exists y \quad I(y) \wedge Y(Gonzalez, y)) \wedge$$
$$(S(Arturo) \rightarrow \exists y \quad I(y) \wedge Y(Arturo, y)) \ \ldots$$

$$\begin{aligned} (S(Arturo) \quad \rightarrow \quad & ((I(Gray) \wedge Y(Arturo, Gray) \vee \\ & (I(Emre) \wedge Y(Arturo, Emre) \vee \\ & (I(Lakdawala) \wedge Y(Arturo, Lakdawala) \vee \\ & (I(Joshi) \wedge Y(Arturo, Joshi) \ \ldots \ ) \end{aligned}$$

Functions

A function is a relation which takes one or more objects of a set, and returns a unique object of a set.

For example, if the domain is ODU students, then we can define a function that takes a student and return the student's advisor.

ad(x) - is the advisor of x

Functions

Functions allow us to make definitions more compact and readable.

Every person is younger than his/her mother.

Y(x,y) - x is younger than y
M(x,y) - x is the mother of y

$\forall w \ \forall z \ M(w,z) \rightarrow Y(z,w)$

let m(x) - mother of x

$\forall w \ Y(w, m(w))$

The Predicate Equality

We can define a predicate,

EQ(x,y) - x is the same object as y.

which is true when x and y are the same objects and false otherwise.

We give this predicate a special symbol:

=(x,y) - x is the same as y.

And use it as an infix:

$x = y$

Homework

Using variable, constants, predicates, functions and quantifiers, represent the follwing statements:

1. My brother is my fathers son.
2. Ed and Kumar have the same grandmother.
3. All mothers have a mother.
4. All mothers have children.

(Hint: define a unary and binary predicate on mothers)

5. Some people have no children.
6. Not all birds can fly.