Inferring Definitive Counterexamples Through Under-Approximation

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Motivation

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- proof absence of bugs using static analysis

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- recover from precision loss due to over-approximating domains

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  - considered invaluable for fixing defects
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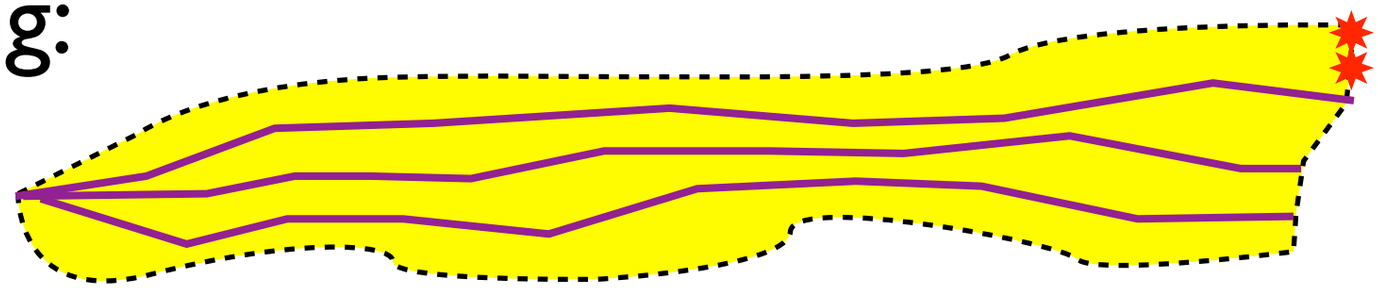
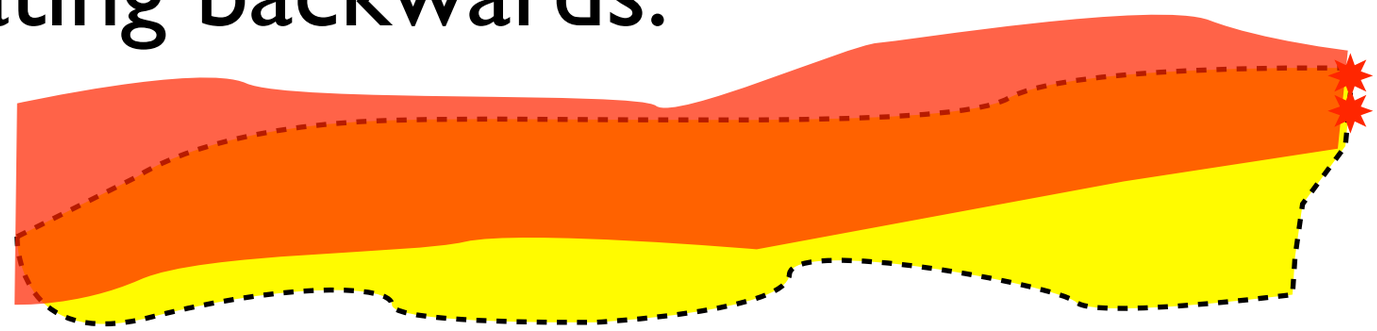
- abstract interpretation merges traces
  - a warning may always be spurious

Revert Merge of Traces

- Spurious warning:



- Over-approximating backwards:



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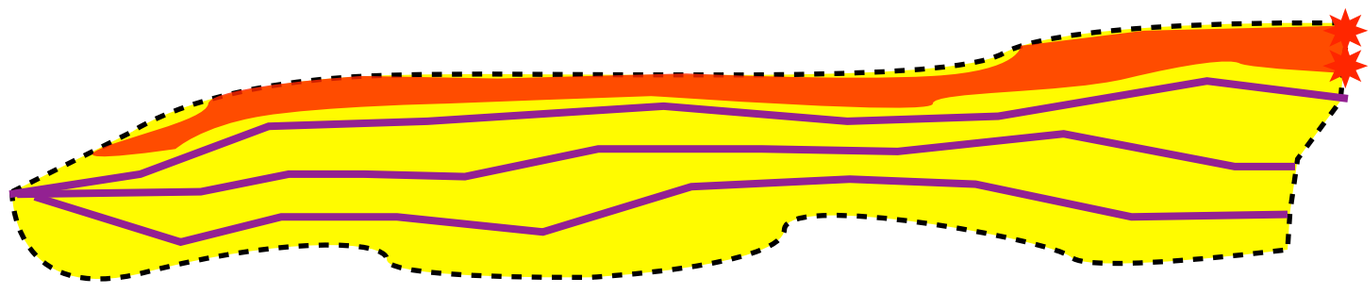

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- Need to solve abduction problem:
  - Given B and C, compute A s.th. $A \wedge B \vdash C$
  - A is called pseudo-complement of B relative to C
- Few domains allow to allow this precisely
  - any domain closed under negation
  - standard numeric domains: no (convexity)

Domains for Backward Reasoning

- Idea: use Domain of Boolean functions
- Express program semantics at the bit-level
- use a relational (input/output) semantics
- `XOR a b` gives $\bigwedge_{i=0}^{31}(\mathbf{a}'[i] \leftrightarrow \mathbf{a}[i] \oplus \mathbf{b}[i])$
- given a post-state `s'` over `a'`, calculate a pre-state `s` over `a`, `b` using SAT-solving

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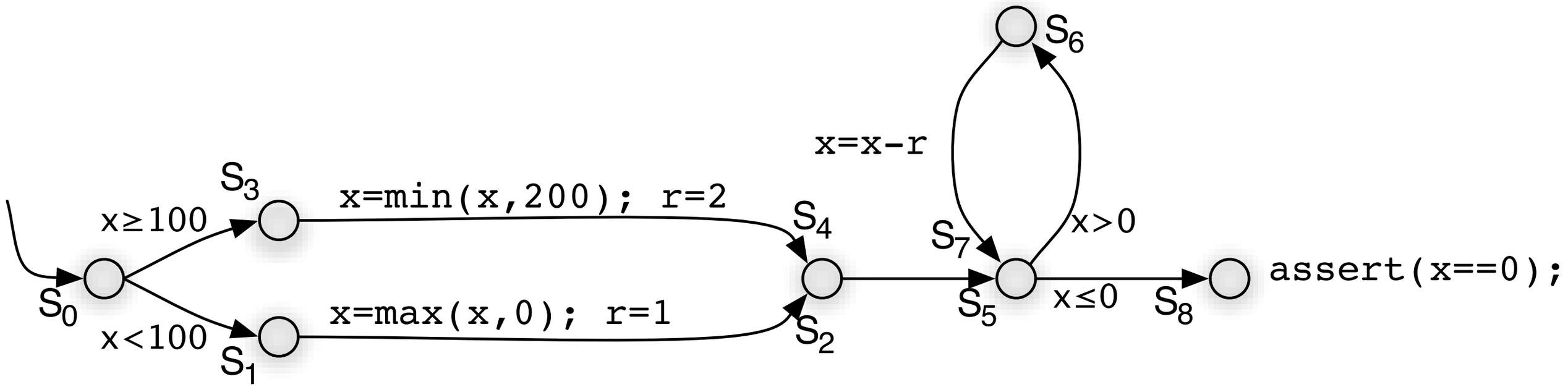
- φ only represents one step: repeat!

Analysis of Loops



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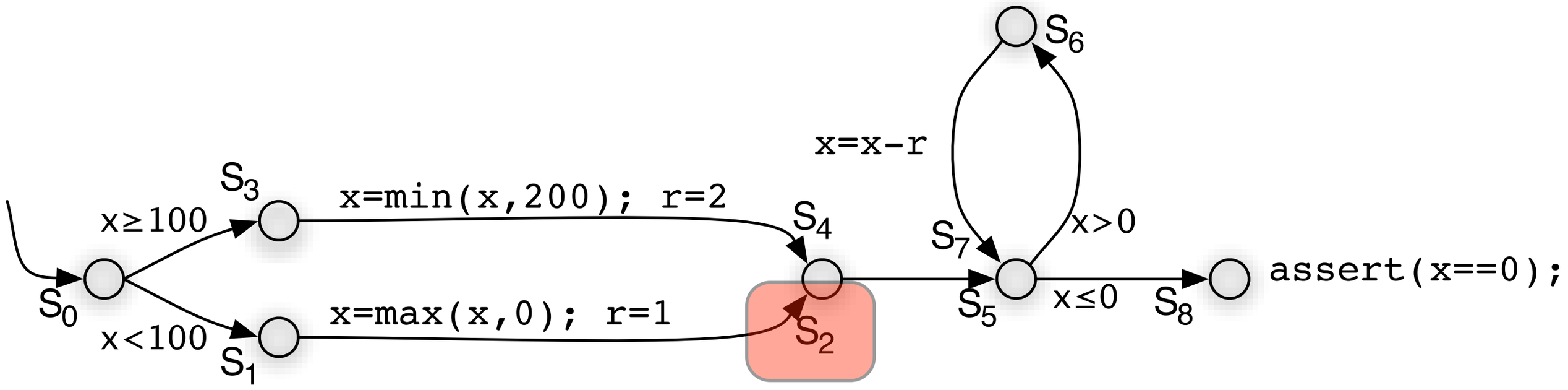

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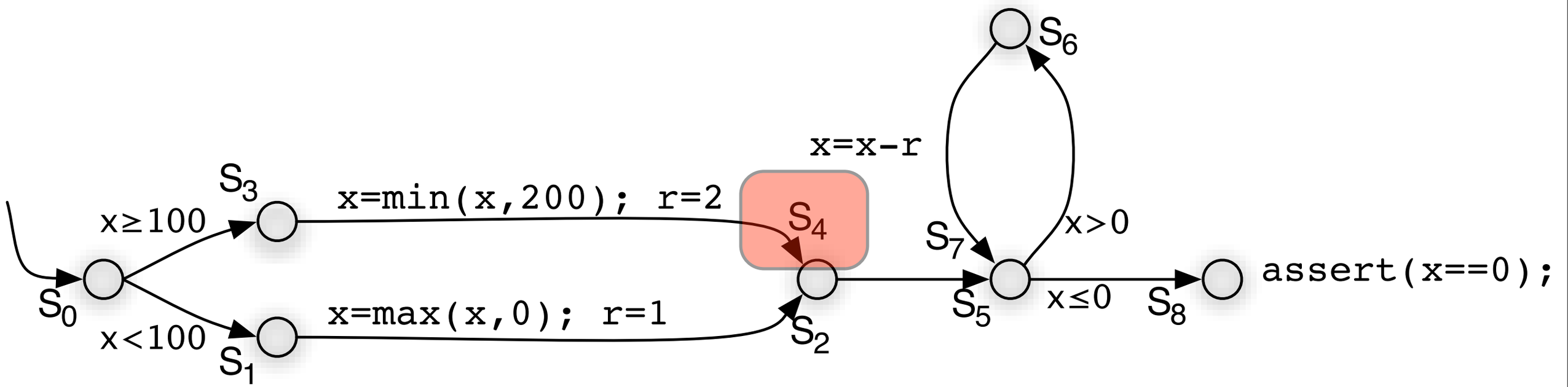

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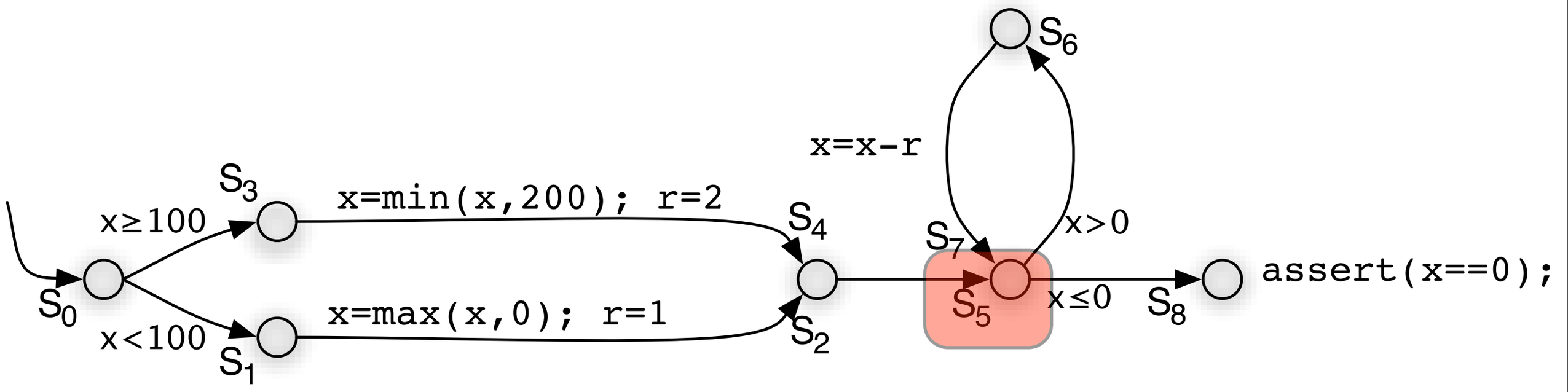

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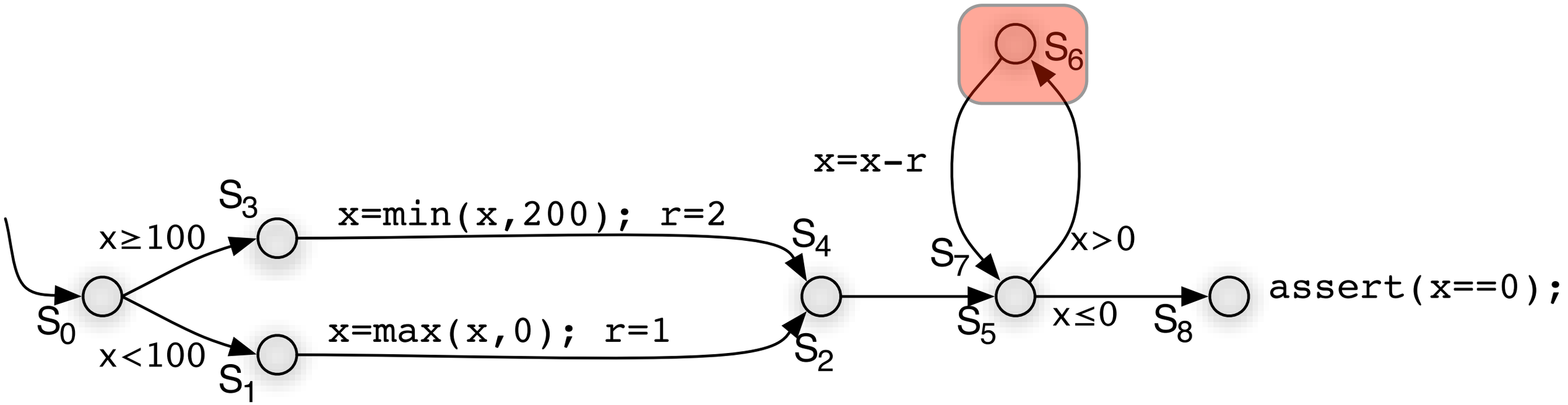

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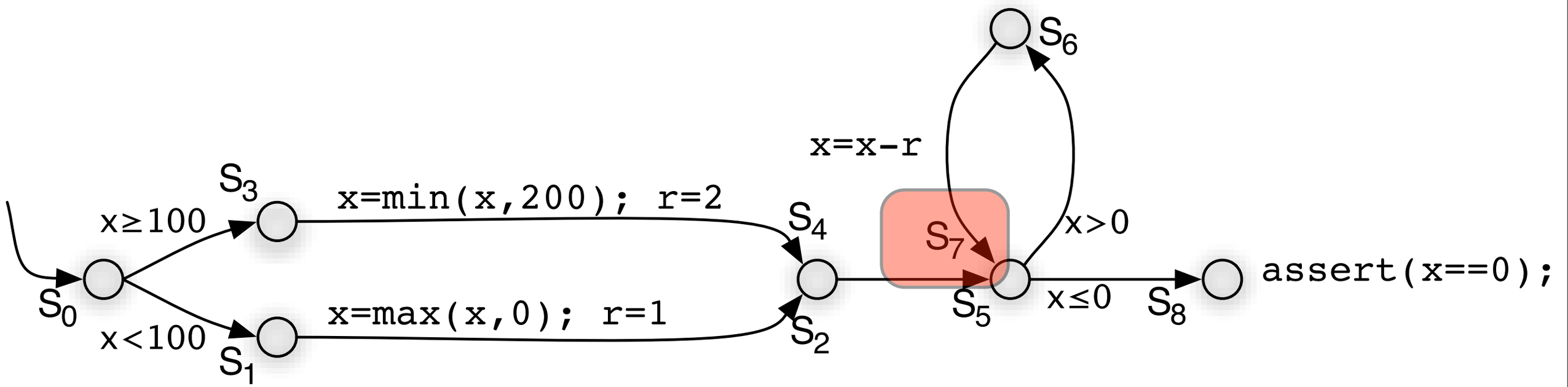

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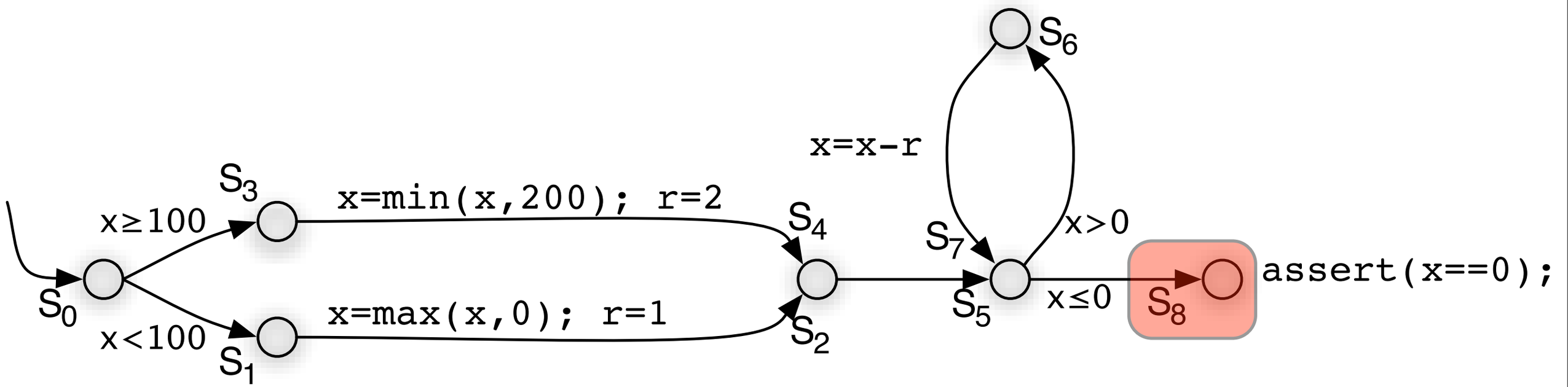

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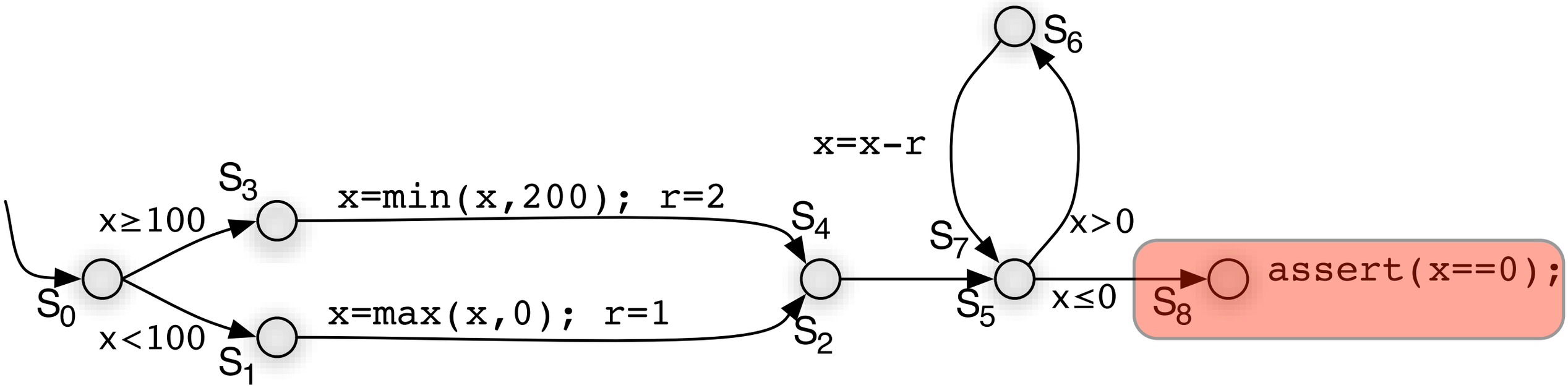

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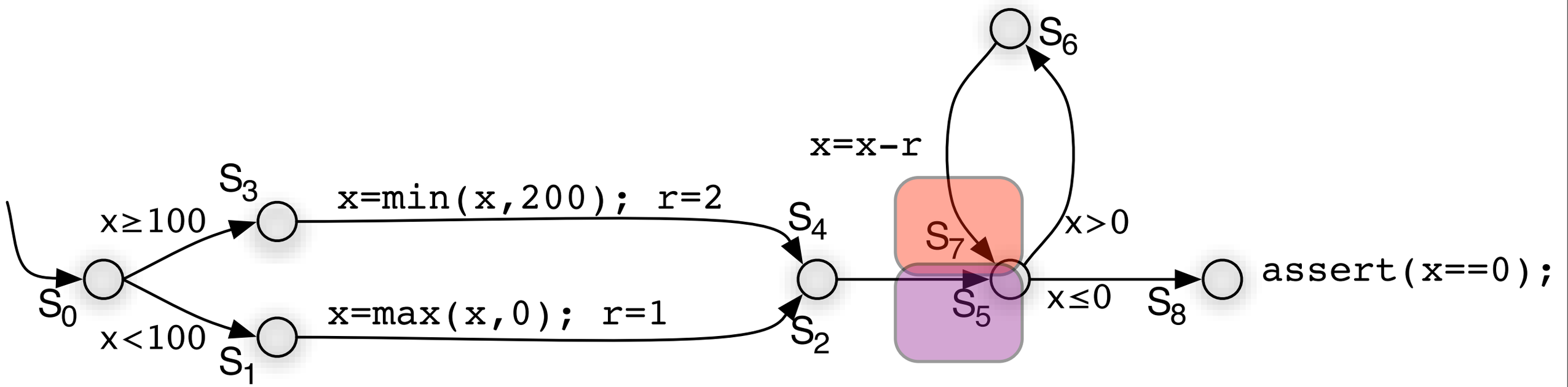

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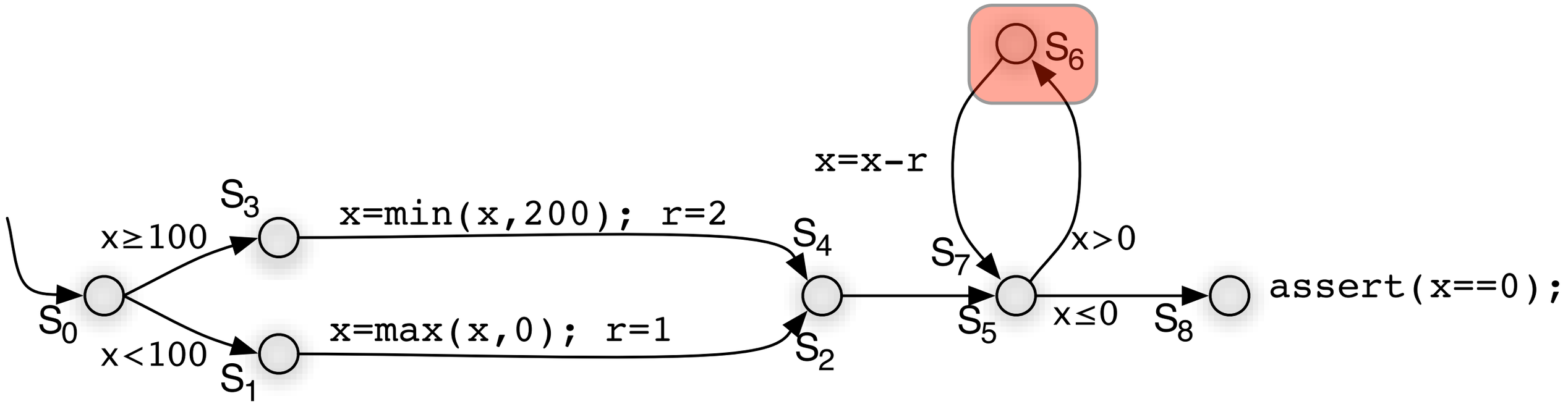

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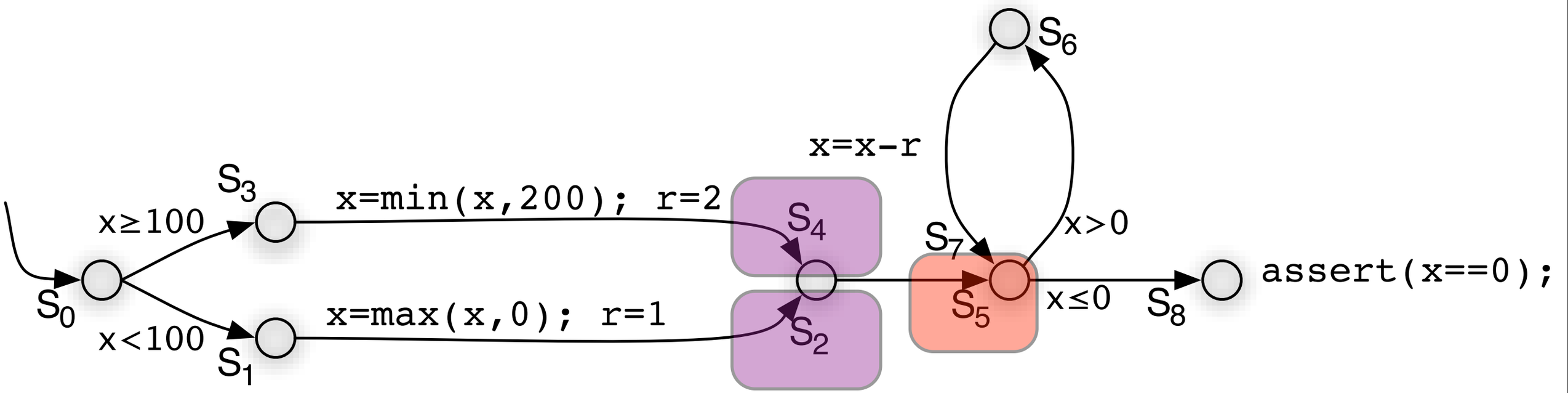

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  - $\varphi_5(S_5) = [\![x = 2n-1, n \in [1,63], r = 2]\!]$

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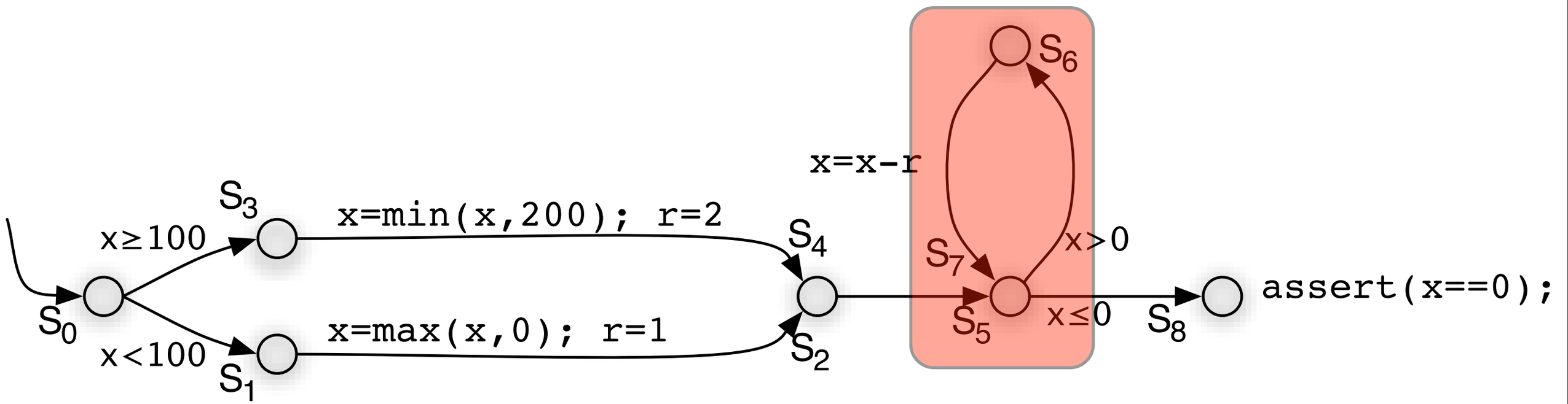

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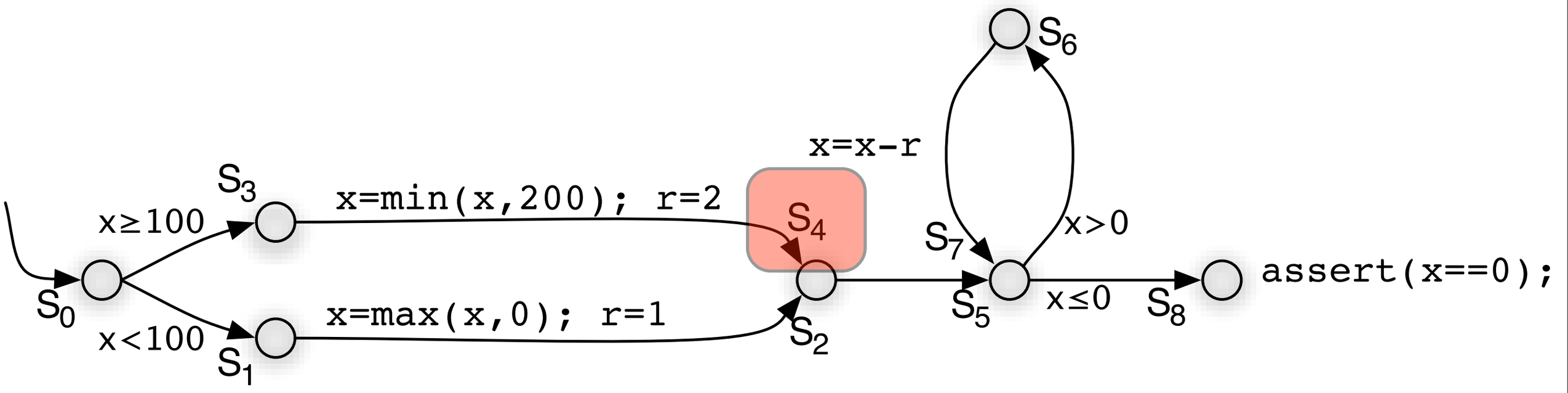

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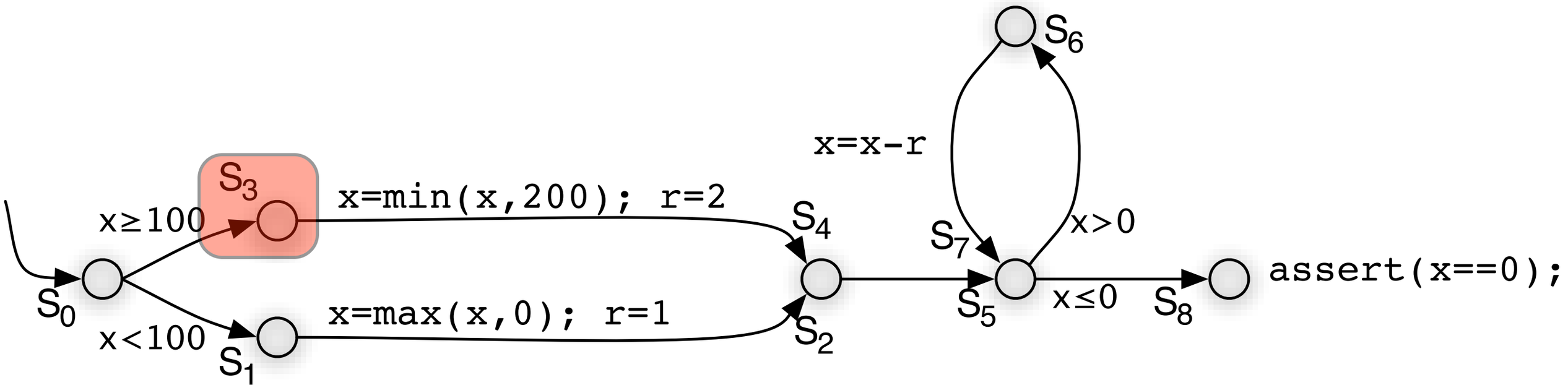

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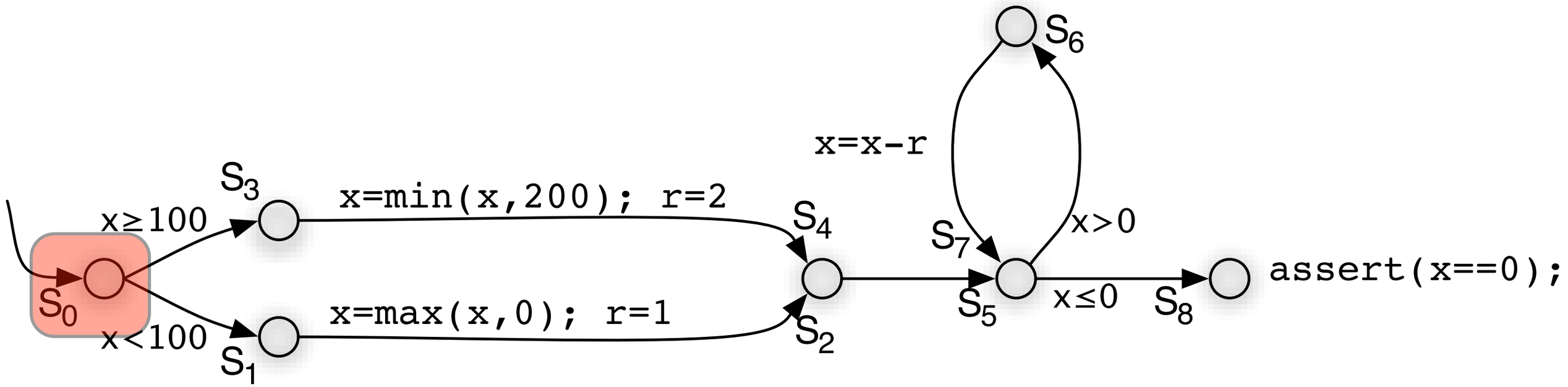

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- if a loop must be unrolled at least 2^i times:
  - calculate $\varphi_{i+1} = \varphi_i \cdot f^{2^i}$
  - instead of $\varphi_{i+1} = \varphi_i \vee \varphi_i \cdot f^{2^i}$

Remove Spurious Warnings

- there exists an `i` such that $\varphi_{i+1} \not\vdash \varphi_i$
- in this state, the warning is spurious:

```
 1  unsigned int log2(unsigned char c) {
 2          unsigned char i = 0;
 3          if (c==0) return 0; else c--;
 4          while (c > 0) {
 5                  i = i + 1;
 6                  c = c >> 1;
 7          }
 8          assert(i <= 7);
 9          return i;
10  }
```

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- f^{2^i} requires formula of size $O(2^i)$
- use projection: $f^2 = \exists_{vars(s')} (f \cdot f)$

Implementation of ∃

- algorithm of [Brauer et al., CAV'11]:
  - given a CNF: $c_1 \wedge \ldots \wedge c_n$,
  - for a subset of variables:
  - calculate d_1, $d_1 \vee d_2$, ..., $d_1 \vee \ldots \vee d_m$ with $d_1 \vee \ldots \vee d_m \models \ldots d_1 \vee d_2 \models d_1$
  - once DNF is complete, apply alg. again

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- could stop calculation of DNF at depth m
- iterative deepening by increasing m?

Incremental Backward Analysis

```
unsigned int hamDist(int x, int y) {
        unsigned int d = 0;
        unsigned int v = x ^ y;
        while (v != 0) {
                d = d + 1;
                v = v & (v - 1);
        }
        assert(d < 32);
        return d;
}
```

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- in example: try $2^5 \leq d \leq 2^6$, $2^6 \leq d \leq 2^7$,...

Implementation

- affine loop transfer function: SAT4J
- projection: MiniSat for CNF→DNF
- BDD for DNF→CNF using CUDD 2.4.2
- tracked times for
  - loop transfer function synthesis (TF)
  - counter example generation (CE)

Experiments

Benchmark	# Instr.	Runtime (Full)		Runtime (Simp.)	
		TF	CE	TF	CE
bit-cnt	26	4.1s	0.9s	0.4s	0.4s
ham-dist	19	4.8s	1.7s	0.8s	0.3s
inc-lshift	14	3.2s	2.7s	0.8s	0.6s
log	22	1.9s	1.3s	0.3s	0.3s
parity	28	8.3s	1.2s	1.2s	0.4s
parity_mit	17	6.2s	2.6s	1.5s	1.2s
randerson	23	8.0s	2.4s	4.2s	0.6s
swap	15	5.9s	1.8s	0.9s	0.5s
loops	207	43.6s	8.0s	13.1s	5.8s

Related Approaches

- CEGAR: counter example in the model is given by model checker
  - need to find trace in concrete semantics
- bounded model checking: limit unrollings
  - we refine/unroll back to front
- backward over-approximation

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  - allow for user input

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  - combine with domains that are able to express abduction (SMT?)
  - allow for user input
- lose soundness: allow for qualified traces?